Targeted advertising and social status

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This paper explores why firms may advertise high-end goods to a broad public, many of whom cannot afford them. Consumers value social status, which depends on what other consumers believe about their wealth. The firm uses informative advertising so that consumers recognize the good when others buy it, which promotes conspicuous consumption. In equilibrium, the firm sells to wealthy consumers but also advertises to poorer consumers at a price they cannot afford. Doing so ensures they understand what the goods signals, which increases the willingness to pay of those who buy. Trade may decrease social welfare, and in particular tends to make poorer consumers worse off.

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Targeted Advertising and Social Status - Draft *

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1 Introduction
Firms sometimes advertise high-end goods to a broad public, many of whom cannot afford to buy them. One example is advertising for cars. Audi spent six million dollars to advertise its $118 000 R8 during the broadcast of Super Bowl XLII, reaching almost one hundred million viewers.1 Prior to the 2008 Formula 1 Canada Grand Prix, Honda showcased its $100 000 Acura NSX at a popular street festival attended by hundreds of thousands of visitors.2

Advertising for clothes provides similar examples. The first three selections in Vogue magazine’s 2008 fall fashion section were a $1200 trenchcoat, a $5500 watch and $600 shoes. Handbags cost between $1700 and $3300.3 Twenty out of thirty-five items from Elle’s fall fashion section cost over $700, including a Peacock feather skirt for $2500.4 Both are mass circulation magazines, with a readership of approximately one million.

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1Brandweek, 12/15/2008, Vol.49, Issue 44 p6-6
2www.newswire.ca/en/releases/archive/June2008/03/c7902
3www.style.com/trendisshopping/theshopper/082008/
4www.elle.com/fashionspotlight
Similarly, large advertising campaigns made Nike Air Jordan shoes and the Apple iPhone household names, even though they were both mainly competing with high-end brands.5

A natural question is why firms might advertise in this way. On the surface, broad advertising for high-end goods appears wasteful. Firms could not reasonably expect most consumers to buy their products in the above examples. It would seem more efficient to target advertising at a smaller group of consumers who are more likely to buy.

After all, firms often do use targeted advertising, putting great effort into selecting which of distinct audiences to reach via specialized cable television, satellite radio, magazines and internet homepages (Esteban, Hernandez, and Moraga-Gonzalez 2006). Targeting technology also continues to improve. Different households watching the same program on cable tv may simultaneously receive different ads, internet service providers can now directly track which websites a person visits, and search engines such as Google and Yahoo auction off space for internet ads conditional on a person’s exact search query (Johnson 2009).

This paper puts forward an explanation for non-targeted advertising based on two ideas. First, consumers value social status, which depends on what other consumers believe about their wealth. Second, informative advertising not only allows a consumer to buy a good, but also to recognize the good when others buy it. For example, consumers who see an iPhone but who are uninformed about it would just believe it is a normal phone. Those who have received an ad would instead appreciate it as a high-end good. Through this mechanism, broad advertising can promote conspicuous consumption by allowing consumers to signal to one another through their purchases.

Section 2 describes the model, where a monopolist faces a market of consumers who are initially uninformed about the good it sells. Consumers differ in their wealth, and each consumer want others to believe he is wealthy. The firm can inform consumers by sending them an ad, which allows them to buy and recognize the good.

Section 3 analyzes a baseline case where the firm sells only one variety of the good. Higher wealth translates into higher willingness to pay, so in equilibrium the firm has an incentive to advertise and sell to wealthy consumers. Buying the good signals above average wealth, which increase the social status of consumers who buy. Status effects therefore allow the firm to charge a higher price, but only to the extent that other consumers recognize the good and understand what it signals. That gives the firm an incentive to advertise to poorer consumers who do not buy, to increase the willingness to pay of wealthier consumers who do.

The mechanism is consistent with the fact that the examples of non-targeted advertising are all for visible goods often associated with social status: cars, clothes, and portable technology. These are goods that others can easily see after purchase. It is widely recognized that visibility is necessary for consumption to

be conspicuous, since only then can it influence observers’ beliefs (Veblen 1994, Frank 1985, Ireland 1994). The twist here is the idea that physical visibility is not enough. Observers must be able both to see the good and to recognize it for what it is.

The mechanism also suggests why firms may sometimes use “lifestyle branding”, where they portray the lifestyle associated with a certain good. Many ads do not transmit detailed product information (Anderson and Renault 2006). For non-targeted ads designed to increase a good’s signaling value, this information is unnecessary. The firm just wants consumers to recognize the good, and to be able to infer what type of people buy it. In the model, consumers anticipate the equilibrium and hence it is enough that they recognize the good. In reality, the firm may also want to explicitly suggest that people who buy the good are, say, wealthy or sophisticated.

After looking at the baseline case, I allow the firm to sell multiple varieties. Section 4 derives the equilibrium when concerns for status are small. I assume each consumer can only buy or recognize a particular variety if he has received an ad for it. In equilibrium, the firm divides the market into segments and sells a different variety to each segment. It advertises each variety to consumers who buy it and to all poorer consumers who do not, but not to wealthier consumers.

Poor consumers receive more ads than wealthy consumers, and are thus better informed about the different varieties. Being better informed does not make poorer consumers better off. All but one of the ads they receive are for varieties they cannot afford.

Any given consumer is also better able to distinguish between consumers who are wealthier than he is, than between those who are poorer. Each consumer receives ads for all varieties bought by wealthier consumers. He can recognize consumers who have about the same wealth as he does, those a bit wealthier, and so on up until the wealthiest consumers. He does not receive any ads for varieties bought by poorer consumers, so he can just form a single belief about them.

It may seem curious that the firm advertises each variety to consumers who do not buy it, even if that variety does not signal particularly high wealth. The intuition is that consumers who buy the highest-end variety want to reveal themselves as having high wealth, so the firm can increase the price if its ads advertise to other consumers. Doing so decreases the status of less wealthy consumers, as everyone now realizes they are not the wealthiest, but it does so regardless of their purchasing decision. Their willingness to pay is therefore unchanged. Consumers who buy the second highest variety then want to reveal themselves as the wealthiest of all remaining consumers. The firm can increase the price by advertising the second highest variety to other consumers, and so on.

The firm does not advertise any variety to wealthier consumers than those who buy it, because doing so would reduce its ability to price discriminate. Consumers who buy a high-end variety are charged a high price, and so would prefer to buy a lower-end variety if they were informed about it.

Section 5 gives an informal discussion of what the type of information the
firm may want to include in its ads. Section 6 looks at welfare for the baseline case, and shows that some consumers would be better off if there was no trade in the status good. That is the case for all consumers who do not buy the good, who suffer a loss of status, but also for some consumers who buy. Trade makes each consumer’s outside option less attractive, since not buying the good now means being revealed as having below average wealth.

I then make specific functional form assumptions and show that when status concerns are small, trade increases consumer welfare. Moreover, the amount of advertising then is socially optimal. When status concerns are large there is too much advertising, and the socially optimal amount of trade would be none at all.

Before presenting the model, I briefly discuss a number of other possible reasons why firms may not use targeted advertising. A first is that advertising technology is imperfect, so that targeting is simply not possible. That is not a particularly convincing explanation for the above examples where the lack of targeting is quite extreme, particularly given the extensive targeting technology available to firms today.

A related reason is that perfect targeting may be too costly. The cost of advertising differs in different media, and the cheapest way to reach a target market might be to advertise in media with a broader reach. The ads then reach consumers who would not buy the good, but from the firm’s perspective that is just a side effect. Hernandez-Garcia (1997), Esteban et al. (2001), and Esteban et al. (2006) look at how this cost reason may cause firms not to target. Their conclusion is that under quite general circumstances, targeted advertising is still optimal.

Another type of explanation is related to anchoring. A consumer’s willingness to pay for a good might increase if he also knows about a similar good which is more expensive; the original good now seems like a “better deal”. More generally, the utility from making a particular choice may depend on the salient available alternatives (Swinkels and Samuelson 2006). This explanation seems plausible in some cases, but it cannot explain why a firm advertises to consumers it does not expect to buy any of its goods.

Finally, firms may use advertising to signal product quality. If consumers are unsure of quality, a firm may have an incentive to burn money through wasteful advertising (Nelson 1974, Milgrom and Roberts 1986). This type of advertising can signal high quality if it is only profitable for a firm that expects repeat purchases, and if consumers only return if they discover that quality is high.

Just as in this paper, advertising allows consumers to infer something valuable about the good. There it is unobserved quality, and consumers must know the firm’s advertising expenditures to understand the signal. Here it is social value related to status, and consumers must know who has received which ads to calculate the status utility from a particular purchase. A major difference is that here the firm does not use advertising to signal to consumers, but rather to exploit consumers’ desire to signal to each other.

To my knowledge, this is the first paper to look at how informative advertis-
ing and targeting relate to status-seeking behaviour. There is a large literature on status effects and conspicuous consumption. For example, Ireland (1994), Ireland (2001) and Bagwell and Bernheim (1996) also take the approach that status depends on beliefs about wealth, consumption or type, rather than on relative position. None of these papers look at the role of advertising.

This paper is also related to the literature on targeted advertising. Hernandez-Garcia (1997), Esteban et al. (2001), and Esteban et al. (2006) all look at monopolists who are able to target high valuation consumers. They conclude targeted advertising increases profits when it is the most cost-effective way to reach consumers who would buy the good, which is usually the case. These papers do not look at consumer status-seeking.

There are a few papers that address the link between advertising and social concerns. Pastine and Pastine (2002) and Clark and Horstmann (2005) show that advertising can help people coordinate on buying a specific good in the case of consumption externalities, for example because of social prestige. The most similar paper is Krahmer (2006), where firms advertise so that ‘the public’ can recognize brand names. Consumers are already fully informed, and advertising just changes the status they receive from the public when buying a good. None of these papers deal with informative advertising or targeting.

2 The Model

A monopolist faces a market of \( n \) consumers, divided equally into \( t \) types (\( n \) large, \( \frac{n}{t} \in \mathbb{Z}^+ \)). Wealth is increasing linearly in type between lower bound \( w_L \) and upper bound \( w_H \), and type is private information. Let \( N = \{1, \ldots, n\} \) be the set of consumers, indexed by \( i \), and \( T = \{1, \ldots, t\} \) be the set of types, indexed by \( j \). A consumer of type \( j \) has wealth:

\[
    w_j = w_L + \left( \frac{j}{t} \right)(w_H - w_L)
\]

The firm produces \( m \geq 1 \) varieties of a status good at constant marginal cost, normalized to zero, where \( m \) is exogenous. In the baseline model analyzed first the firm will only produce one variety, so in that case \( m = 1 \).

When \( m \geq 2 \), varieties are similar in the sense that each gives the same intrinsic utility to consumers. Varieties will only differ in their signaling value which arises in equilibrium. Let \( M = \{x_1, \ldots, x_m\} \) be the set of varieties, indexed by \( k \).

The firm moves first by deciding, for each variety \( x_k \), to which types of consumers it will advertise, \( a(x_k) \), and by committing to a single price \( p(x_k) \). The firm’s advertising technology allows it to advertise each variety to as many types as it likes, or to none at all.

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6Advertising does inform the public, but the author argues explicitly against interpreting it as informative advertising. The reason is that it does not inform consumers and only helps the public recognize brand names.
The firm’s strategy is therefore $s_f = (A, P)$, where $A = (a(x_k))_{x_k \in M}$, $a(x_k) \subseteq T \cup \{\emptyset\}$ and $P = (p(x_k))_{x_k \in M}$, with $p(x_k) \in \mathbb{R}^+$. Write $a_k \equiv a(x_k)$ and $p_k \equiv p(x_k)$ for short.

I do not want cost considerations to drive the firm’s advertising decision, so I assume advertising is costless. As a tie-breaking rule, the firm strictly prefers one strategy over another if it yields the same revenue but involves strictly less total advertising $\sum_{x_k \in M} |a_k|$.

Consumers are initially uninformed about each variety. If the firm advertises variety $x_k$ to type $j$, then it informs all consumers of that type about that variety. Informing a consumer about variety $x_k$ does not inform him about any other variety $x_{k'}$ with $k' \neq k$. Advertising is purely informative.

Consumers respond to the firm’s advertising and pricing by making a purchasing decision. Each consumer can buy zero or one unit of the status good, and only of a variety of which he is informed. He spends any remaining income on a composite good. The price of the composite good is normalized to one.

Let $B_j$ be the set of varieties consumer $i$ of type $j$ can afford and of which he is informed; $B_j = \{x_k | j \in a_k, p_k \leq w_j\}$. The consumer’s strategy is a function $s_i : B_j \rightarrow B_j \cup \{\emptyset\}$. Denote his chosen action by $\alpha_i$, where $\alpha_i = x_k$ if he buys variety $x_k$ and $\alpha_i = \emptyset$ if he only buys the composite good.

Defining $p_\emptyset \equiv 0$, the firm maximizes profits:

$$\pi = \sum_{i \in N} p_{\alpha_i}$$

Consumer preferences are additively separable in intrinsic utility $U_I$ and status utility $U_S$. A consumer $i$ of type $j$ who takes action $\alpha_i$ has intrinsic utility:

$$U_I = V'(w_j - p_{\alpha_i}) + u_01_{\alpha_i \neq \emptyset}$$

The first term is the utility from the composite good, with $V' > 0$ and $V'' < 0$. The second term is the intrinsic utility from the status good, $u_0$, which is the same for each variety.

The consumer’s status utility depends on what other consumers believe about his wealth. His status utility is:

$$U_S = \lambda_j \sum_{e \in N \setminus i} \mu_{i'}(w_{i'} | (A, \times_{\nu' \in N} \alpha_{i'}))$$

Status utility depends on the average beliefs of all other consumers, where consumer $i$’s beliefs are $\mu_{i'}$. These beliefs depend in turn on each consumer’s purchasing decision and on the firm’s choice of advertising $A$. In particular, consumer $i$’s beliefs depend on whether he is informed about the variety bought by consumer $i$.

The term $\lambda_j > 0$ gives how much type $j$ cares about status, which may vary with $j$. I assume $\lambda_j$ is monotonic, and it will be constant in the baseline case. The term $N - 1$ in the denominator from taking the average over beliefs is incorporated into $\lambda_j$. 

6
The beliefs of any consumer \( i' \) about consumer \( i \) follow from Bayes’ rule. Consumer \( i' \) conditions his beliefs on the information at his disposal in the following way. If consumer \( i \) buys a variety of which \( i' \) is informed, then \( i' \) recognizes that variety and believes \( i \) is the expected type of someone who buys that variety. If consumer \( i \) does not buy a variety of which \( i' \) is informed, then \( i' \) does not recognize that variety and believes \( i \) is the expected type of someone who does not buy a variety of which he is informed. In the later case, he believes \( i \) either buys a variety of which he is uninformed, or only buys the composite good. Consumer \( i' \) cannot condition his beliefs on the amount \( i \) consumes of the composite good, which in that sense is non-visible.

When I look at welfare, it will be appropriate to normalize status utility so that it depends on beliefs about \( w - (w_L + w_H)/2 \), so about how a consumer’s wealth differs from average wealth in society. The normalization would not affect the equilibrium analysis, as it just corresponds to adding a constant to each consumer’s utility function.

To summarize, I look for a Bayes-Nash equilibrium, consisting of strategies \((s_f, x_i \in N s_i)\) and beliefs \(\mu\) about each consumer’s type after he has made a purchasing decision. Each player’s strategy is a best response to the strategies of the others, given beliefs. Equilibrium beliefs follow from strategies via Bayes’ rule. There is no a priori restriction on beliefs about a consumer who makes a purchasing decision that does not occur in equilibrium. I look for a symmetric equilibrium where all consumers of the same type take the same action.

### 3 Analysis - Baseline Case

The number of consumers \( n \) is large, so I can assume each consumer does not take his own type into account when forming beliefs about others. Because I look at symmetric equilibria, I can work with \( n = t \) effective consumers, one per type, and identify each effective consumer’s equilibrium action with that of all consumers of his type. I use \( j \) to index both consumers and types, so consumer \( j \) is of type \( j \) and has wealth \( w_j \).

Consider the baseline case where \( m = 1 \) and \( \lambda_j = \lambda \) for all \( j \). The firm produces only one variety of the status good and consumers all care about status to the same extent. I first calculate a consumer’s willingness to pay for the status good, given that the firm advertises to \( r < t \) consumers at price \( p \) and given the purchasing decisions of other consumers.

Let \( w_{buy} \) be the expected wealth of a consumer who buys the status good

\[
w_{buy} = \frac{\sum w_j \chi_{\alpha_j = 1}}{\sum \chi_{\alpha_j = 1}}
\]

where the summation is taken over all consumers.

If consumer \( j \) buys the status good, then his utility is:

\[
V(w_j - p) + u_0 + \lambda r(w_{buy}) + \lambda(t - r)(\frac{w_H + w_L}{2})
\]
The other informed consumers recognize consumer $j$ has bought the status good, and thus believe he has expected wealth $w_{buy}$. The uninformed consumers cannot distinguish him from those who only buy the composite good. Their belief is just the prior, that he has average wealth $(w_L + w_H)/2$.

If consumer $j$ does not buy the status good, his utility is:

$$V(w_j) + \lambda r(w_{not}) + \lambda(t - r)(\frac{w_H + w_L}{2})$$

where $w_{not}$ is the expected wealth of someone who only buys the composite good, and is defined in an analogous way to $w_{buy}$.

Consumer $j$’s willingness to pay for the status good is therefore the minimum of his wealth $w_j$ and the value of $p$ that satisfies:

$$V(w_j) - V(w_j - p) = u_0 + \lambda r(w_{buy} - w_{not})$$

Denote that willingness to pay by $v(j)$. Rearranging gives:

$$v(j) = \min \{w_j, w_j - V^{-1}[V(w_j) - u_0 - \lambda r(w_{buy} - w_{not})]\}$$

I will assume that $V$ and parameters are such at $w_j < v_j$ for each consumer. For example, that will always be the case if $V(w) = \ln(w)$, or if $w_L$ is large enough relative to the other parameters. I do so to avoid complications from a consumer wanting to pay more for the status good than his total wealth, which would not yield more insight. Willingness to pay is therefore:

$$v(j) = w_j - V^{-1}[V(w_j) - u_0 - \lambda r(w_{buy} - w_{not})]$$

The status term in willingness to pay depends only on the beliefs of the $r$ informed consumers. The beliefs of uninformed consumers affect consumer $j$’s utility, but they are independent of his purchasing decision.

The concavity of $V$ implies that $v(j)$ is increasing in $j$. Wealthier consumers have a strictly higher willingness to pay for the status good because of their lower marginal utility of wealth. That is the case regardless of the value of $(w_{buy} - w_{not})$, since all consumers care about status to the same extent.

If the firm serves the whole market, then $w_{not}$ in the above expression is not defined. Willingness to pay will depend on the out-of-equilibrium beliefs about a consumer who only buys the composite good.

In the absence of status effects, the firm would just solve the standard problem of a monopolist facing a downward sloping demand curve. The demand curve would be determined by consumer willingness to pay, given by the above equation for $v(j)$ with $\lambda = 0$. Clearly, the firm would only advertise to those consumers who would buy the good.

The firm’s optimal strategy in the presence of status effects differs in that it also advertises to all less wealthy consumers who do not buy the status good.

**Theorem 1.** Let $n = 1$ and $\lambda_j = \lambda$ for all $j$. Then the firm chooses $s_f = (a, p)$ with $a = \{1, \ldots, T\}$ and $p = v(j_0)$ where $j_0$ is the lowest type such that:
\[(t - j_0 + 1)v(j_0) \geq (t - j_0 + 2)v(j_0 - 1)\]

and \(j_0 = 1\) if there is no such type.

\[p = v(j_0) = w_{j_0} - V^{-1}[V(w_{j_0}) - u_0 - \lambda(t\frac{w_H - w_L}{2})]\]

Consumers choose \(\alpha_j = x_1\) iff \(j \geq j_0\).

Proof. See appendix \(\square\)

In words, the firm sets the price so that a critical type \(j_0\) is indifferent about buying, and sells the status good to all consumers above that critical type. Denoting the wealth of consumer \(j_0\) by \(w_0\), it sells to all consumers with wealth \(w \geq w_0\). It advertises the status good to all consumers.

The intuition for the result is as follows. For any given quantity sold, the firm wants to sell to as high types as possible. These consumers have a higher willingness to pay, and each one who buys makes the status good send a better signal. The firm therefore sells only to consumers above a critical type. If the firm does not sell to the whole market, then buying the status good sends a strictly better signal than not buying it. The firm can advertise to all consumers to ensure that everyone understands the signal.

The result reflects the intuition described in the introduction. The firm advertises the status good to those who buy it, but also to less wealthy consumers who are unwilling to buy at the advertised price. Doing so increases the status utility of consumers who buy the good, which lets the firm charge a higher price.

Perhaps surprisingly, a consumer’s incentive to buy the status good does not depend on exactly how exclusive it is. It is important that the status good be somewhat high-end, in the sense that only consumers with type above some critical value buy it. However, a consumer’s incentive to buy the status good does not depend on the firm’s choice of critical value.

If the firm chooses a high critical value, then only very high types buy the status good. Consumers are willing to pay a high price because buying the good shows they are very wealthy. Not buying only signals their wealth is slightly below average.

If the firm chooses a low critical value, then many different types buy the status good. Buying only signals a consumer’s wealth is slightly above average. But consumers are still willing to pay a high price, because not buying the good shows they have very low wealth.

By just looking at consumers’ actions in these two different situations, it may seem in the former case like consumer’s are trying to differentiate themselves from others, and in the later case like they are trying to conform. Here, either situation can occur just because consumers care about direct beliefs about their own wealth.

To set the critical value \(j_0\), the firm faces a similar problem to the case where status effects are absent. The only difference is that consumers behave as if the status good gives them intrinsic utility \(u_0 + \lambda([w_H - w_L]/2)\) rather than just...
The firm sets \( p \), which determines \( j_0 \), by balancing the marginal and infra marginal effects of a change in price.

If the firm sells to the entire market \( (j_0 = 1) \), then willingness to pay depends on the out-of-equilibrium beliefs about a consumer who only buys the composite good. The most natural beliefs are that such a consumer has the lowest wealth \( w_L \). Such beliefs can be justified in the following way. If the firm chose different critical values \( j_0 \) that tended one, then the beliefs implied by Bayes’ rule would tend to \( w_L \). With these out-of-equilibrium beliefs, a consumer’s willingness to pay for the status good is the same for \( j_0 = 1 \) as it is for any \( j_0 \geq 2 \).

The same type of equilibrium outcome will hold if \( \lambda_j \) is increasing in \( j \), so if wealthier consumer are more concerned with status. In this situation willingness to pay will increase with type if \( (w_{buy} - w_{not}) > 0 \), which will again give the firm an incentive to sell only to types above some critical value. The same type of outcome will also hold if \( \lambda_j \) is decreasing in \( j \), as long as it is not decreasing by too much. The important thing is that equilibrium willingness to pay \( v(j) \) still be increasing in \( j \). That will be the case if high types’ lower marginal utility of wealth outweighs their lower marginal utility from status.

There would be no role for advertising if all consumers were already informed about the status good, or equivalently if they had zero search costs. In contrast, the following informal argument suggests that in the presence of very low search costs, the firm might have an incentive to only advertise the status good to consumers who will not buy it!

Say advertising was costly and search costs small. High type consumers have an incentive to search, because they anticipate the equilibrium outcome and know that buying the status good will increase their utility. The firm does not need to advertise in order to make sales, and so can save on advertising costs. Low types have no incentive to search because they correctly anticipate they are better off just consuming the composite good. The only way for the firm to inform these lower type consumers, and to take advantage of status effects, is through advertising.

4 Analysis - Multiple Varieties

I now consider the case where the firm sells multiple varieties \( (m \geq 2) \), and where concerns for status \( \lambda_j \) may vary with type. To keep the analysis tractable, I assume that \( \lambda_j \) is small for all \( j \) but still strictly positive. The firm can only sell one variety to each type, so I assume without loss of generality that the number of varieties is no more than the number of types, \( m \leq t \).

If status effects were absent, the firm would just divide the market up into \( m \) segments, one per variety, and carry out price discrimination between the different segments. All varieties would give the same utility, and the price of each variety would make the lowest type to buy it indifferent with only buying the composite good. The firm would only advertise a given variety to consumers who buy it.
Similar to the case of the baseline model, with status effects the firm advertises varieties to less wealthy consumers than those buy them.

**Theorem 2.** Let \( \lambda_j \) be small for all \( j \), and define \( j_m \equiv t, j_{-1} \equiv 1 \). The firm chooses critical values \( j_0, \ldots, j_{m-1} \), and \( s_f = (A, p) \) such that, if \( j_0 \geq 2 \), then \( a_k = \{1, \ldots, j_k\} \) for all \( k \in M \). If \( j_0 = 1 \), then there is instead one value of \( k \) such that \( a_k = \{2, \ldots, j_k\} \). It sets prices:

\[
p_k = w_{j_k-1} - V^{-1}\left\{ V(w_{j_{k-1}}) - u_0 - \lambda_{j_{k-1}} \sum_{k' = 0}^{k} (j_{k'} - j_{k'-1}) \right\}
\]

Consumers of type \( j \) choose \( \alpha_j = x_k \) if \( j_{k-1} \leq j < j_k \), and \( \alpha_j = \emptyset \) if \( j < j_0 \).

In words, the firm divides the market up into \( m \) segments and sells one variety to consumers in each segment. It sells variety \( x_1 \) to consumers with type in \([j_0, j_1 - 1]\), variety \( x_2 \) to consumers with type in \([j_1, j_2 - 1]\) and so on, selling variety \( x_m \) to consumers with type in \([j_{m-1}, t]\). It sets the price for each variety such that the lowest type to buy it is indifferent with only buying the composite good. All this corresponds to the firm’s optimal strategy when status effects are absent.

The firm advertises each variety to consumers who buy it and to all lower types. The only exception is if \( j_0 = 1 \), so if every consumer buys some variety of the status good. In that case, the firm only send ads for \( m - 1 \) of the \( m \) varieties to consumers with type in \([1, j_1 - 1]\). That is enough for these types to identify a consumer who buys any variety. They infer that anyone they do not recognize must have bought the single variety of which they are uninformed.

Just as in the baseline case, the firm advertises varieties to lower type consumers to take advantage of status effects. However, allowing the firm to sell multiple varieties yields a number of new insights.

The firm advertises the highest-end variety \( x_m \) to all lower type consumers who do not buy it, but it also does the same for every variety \( x_k \). It advertises \( x_k \) to all lower types, regardless of whether the average type of someone who buys \( x_k \) is high or low. The idea is that consumers who buy \( x_m \) are the highest types, and they are willing to pay more for that variety if others can understand what \( x_m \) signals. If the firm advertises \( x_m \) to inform all lower type consumers, then the consumers who buy \( x_m-1 \) are the highest remaining types to buy a variety of which some consumers may be uninformed. They are willing to pay more for \( x_{m-1} \) if others can recognize that variety, so that they distinguish themselves from all lower types. Continuing in this way, even consumers who buy the lowest variety \( x_1 \) want to distinguish themselves from even lower types who only buy the composite good. Still, the amount of advertising for a variety \( x_k \) is increasing in \( k \), and the firm uses more non-targeted advertising for higher-end varieties.

Advertising a variety \( x_{k'} \) to all lower types not only increases the willingness to pay of those who buy \( x_{k'} \), but it may also increase the willingness to pay for those who buy a lower variety \( x_k \) with \( k < k' \). It will never decrease the
willingness to pay of the latter group of consumers, though it may decrease their utility.

Take the case of variety $x_m$, and consider some consumer who was uninformed about both $x_m$ and $x_k$. Once that consumer receives an ad for $x_m$, he can distinguish that variety from $x_k$. He downgrades his beliefs about consumers who buy $x_k$, since he now knows they are not among the highest types. That decreases the status utility of those who buy $x_k$. However, he also downgrades his beliefs in the same way about consumers who only buy the composite good, and that is the best outside option for those who buy $x_k$. Consumers who buy $x_k$ in equilibrium are now worse off regardless of their purchasing decision. Their willingness to pay for $x_k$ is unchanged.

Advertising $x_m$ to all lower types is even more advantageous for the firm if lower type consumers are informed about $x_k$ but uninformed about $x_m$. Advertising $x_m$ then leaves the equilibrium beliefs about those who buy $x_k$ unchanged, as consumers can already recognize that variety. But just as above, it reduces the status utility of anyone who take his best outside option and only buys the composite good. Willingness to pay for $x_k$ increases.

The firm does not advertise any variety $x_k$ to higher type consumers than those who buy. That is, non-targeted advertising for a variety goes exclusively to less wealthy consumers. The above argument would suggest that also advertising $x_k$ to higher types would further increase willingness to pay for $x_k$ through status effects. The problem is that advertising $x_k$ to higher types would reduce the firm’s ability to price discriminate. Consumers who buy a higher variety $x_{k'}$ with $k' > k$ have a lower marginal utility of wealth than those who buy $x_k$. That allows the firm to charge a higher price for $x_{k'}$ than it does for $x_k$. If the firm advertised $x_k$ to these higher types, it would have to drop the price of $x_{k'}$ to prevent the higher types from buying $x_k$ instead. Status effects are small, so this negative effect outweighs any increase in willingness to pay for $x_k$ that the advertising may generate through status effects.

One feature of the equilibrium is therefore that lower type consumers receive more ads than higher types. They are better informed, and so better able to distinguish between consumers who buy different varieties. That being said, being better informed does not increase their utility. From the perspective of these lower types, most of the ads they receive are of no use. They advertise expensive varieties that these consumers are unwilling to buy.

Another feature of the equilibrium is that any given consumer is better informed about those who have higher type than he does, than about those who have lower type. The consumer is fully informed about all higher-end varieties than the one he buys. If he buys variety $x_h$, he is able to distinguish between consumers who have about the same wealth as he does (those who buy $x_h$), consumers who are a bit wealthier (those who buy $x_h + 1$), and so on up until the wealthiest consumers who buy $x_m$. In contrast, the consumer is completely uninformed about all lower-end varieties. He is unable to distinguish between consumers who have lower wealth than he does, and just forms a single expectation about this group.

The beliefs of the lowest types have the greatest impact on each consumer’s
willingness to pay for the variety he ends up buying. This is despite the fact that status utility depends only on average beliefs over all consumers. The reason is that the lowest types are most informed about higher-end varieties, and so have the most negative beliefs about consumers who take their outside option. That can be seen by looking at the firm’s optimal price $p_k$ for variety $x_k$, as stated in the theorem.

The firm sells $x_k$ to types in $[j_{k-1}, j_k - 1]$, and sets the price to make type $j_{k-1}$ indifferent with only buying the composite good. As argued above, willingness to pay for $x_k$ is not affected by the beliefs of consumers who buy any higher variety $x_{k'}$ with $k' > k$. That is why the summation in the expression for $p_k$ only runs from 0 to $k$.

Consider lower type consumers who buy some variety $x_{k'}$ with $k' < k$. These consumers are informed about $x_k$, and so believe that those who buy $x_k$ have expected wealth $(w_{j_{k-1}} + w_{j_k - 1})/2$. If a consumer instead only buys the composite good, those who buy $x_{k'}$ believe he is the expected type of someone who does not buy any variety $x_{k'}, x_{k'+1}, \ldots, x_m$. That is, he has expected wealth $(1 + w_{j_{k'-1}-1})/2$, which is increasing in $k'$. The second term in the summation is just the difference between these two expressions, and it is larger for lower types because they are more informed.

5 What information should be advertised?

I have assumed that a firm’s ads for a given variety always transmit two types of information. They allow consumers to recognize the variety when others buy it, and to buy the variety themselves.

In reality, the firm probably has some flexibility in choosing what type of information to transmit through an ad. The mechanism described in this paper would then suggest the firm may not want its ads to be fully informative. I now discuss this point informally.

Take the case of one variety, where the firm advertises the status good to consumers who do not buy it. The firm does so only so that these consumers can recognize the good, and is indifferent about whether they can actually buy it.

It is not implausible that the firm faces a trade-off in what type information it can transmit through an ad. An ad that ensures consumers remember the practical information required to buy the good may differ from an ad that ensures they will recognize the good when they see it. If faced with such a trade-off, however slight, the firm would prefer to use non-targeted advertising that is less informative overall but more informative in the latter dimension.

Now take the case where the firm sells multiple varieties. Say it could send an ad that only allows consumers to recognize a variety, without allowing them to buy it. The firm would then no longer face a trade-off between status effects and price discrimination when advertising a variety $x_k$ to higher types. Sending all higher types such an ad would increase the willingness to pay of consumers who buy $x_k$ through status effects, without reducing the firm’s ability to price
discriminate. The firm would advertise each variety to every consumer, but the informational content of the ads would vary.

The firm might have an incentive to send ads that are even less informative. Say it could send an ad that just allows consumers to distinguish the status good from the composite good, but not to distinguish between different varieties. A consumer who receives such an ad forms the same beliefs about all those who buy the status good. Compared to a fully informative ad, that decreases the status utility of consumers who buy the higher varieties and increases the status utility of those who buy lower varieties. If lower types value status more than higher types, so if the ratio \( \lambda_j / \lambda_{j'} \) is small for \( j > j' \), then the net effect might well be to increase firm profits.\(^7\)

6 Welfare

I now return to the baseline model with one variety and status concerns that do not vary with type \((m = 1, \lambda_j = \lambda)\), and I look at welfare. I compare the equilibrium outcome with what would occur if there were no trade in the status good. The externalities from status effects mean that trade can have a negative effect on welfare.

I first consider consumers of a given type, and look at whether trade in the status good increases their utility. Recall that \( j_0 \) denotes the critical type with wealth \( w_0 \) such that the firm sells the status good to all higher type consumers.

**Theorem 3.** Let \( m = 1 \) and \( \lambda_j = \lambda \). Then there exists \( w' > w_0 \), such that trade in the status good increases the utility of consumers with wealth \( w \) if and only if \( w \geq w' \).

**Proof.** For a consumer with wealth \( w \), take utility with trade in the status good minus utility when there is no such trade in the status good, and differentiate with respect to \( w \). That gives \( V'(w - p) - V'(w) \), which is strictly positive by \( V''w < 0 \). The change in utility is higher for higher types, which implies the existence of a critical value \( w' \).

For any type \( w < w_0 \), the change in utility from trade in the status good is:

\[
V(w) + \lambda\left(\frac{w_L + w_0}{2}\right) - V(w) - \lambda\left(\frac{w_L + w_H}{2}\right) < 0
\]

For type \( w = w_0 \), the change is:

\[
V(w - p) + u_0 + \lambda\left(\frac{w_0 + w_H}{2}\right) - V(w) - \lambda\left(\frac{w_L + w_H}{2}\right)
\]

which is also strictly negative since the firm sets \( p = w_0 - V^{-1}[V(w_0) - u_0 - (w_L + w_H)/2] \)

\(^7\)The assumption that the ratio \( \lambda_j / \lambda_{j'} \) is small for \( j > j' \) need not conflict with the assumption that equilibrium willingness to pay is increasing in type \( j \), as long as each \( \lambda_j \) is small.
There are always some consumers who suffer a loss in utility from trade in the status good, and it is those consumers whose wealth lies below a certain threshold. Moreover, the threshold is higher than the lowest type to buy the status good. Trade hurts all consumers who do not buy the status good, and even some who do.

Consumers who do not buy the status good suffer a decrease in status utility. Not buying when higher types do reveals them as having relatively low wealth. The lowest type consumers to buy the status good gain intrinsic utility \( u_0 \) as well some status utility, but are left worse off because of the high price they pay. The firm charges a price \( p \) that makes these consumers indifferent with their outside option, but trade in the status good makes that outside option worse than it was before. It now involves being revealed as a relatively low type. These consumers are willing to buy the status good given that others do the same, but they would be better off if nobody bought it.

The above results still hold if \( \lambda_j \) is strictly increasing or decreasing in \( j \). The only requirement in the later case is that \( \lambda_j \) not decrease by too much with \( j \), so that the equilibrium outcome is still of the form given in Theorem 1.

I now look at aggregate consumer and total welfare. The concavity of \( V \) means that there are wealth effects, so consumer surplus is not well defined. I instead evaluate consumer welfare in terms of compensating variation (CV). That is, given trade in the status good, I look at what transfer each consumer would need to achieve the same utility as if there was no trade in the status good, and then I sum over these transfers. A consumer’s contribution to the compensating variation is positive if that consumer is made worse off by trade in the status good.

I will say trade in the status good has a negative effect on consumer welfare if \( CV > 0 \), and a negative effect on total welfare if \( \pi - CV < 0 \). One must be careful about how to interpret these statements. Formally, \( CV > 0 \) means trade in the status good followed by carrying out zero sum transfers between consumers cannot lead to a Pareto improvement. \( \pi - CV < 0 \) means that trade in the status good cannot lead to a Pareto improvement even if all firm profits are returned to consumers, as in a general equilibrium framework.

I make the following simplifying assumptions for the subsequent analysis. I assume the that \( \lambda \) is constant, and that \( V(w) = \ln(w) \). That is convenient because it ensures each consumer’s willingness to pay is always strictly less than his wealth, regardless of the value of \( \lambda \). It also implies that the firm’s optimal strategy is always to sell the status good to the same group of consumers. Changes in \( \lambda \) just translate into changes in price. I assume \( w_H > 2w_L \) so the firm does not serve the entire market, and that the number of types \( t \) is large. That means I can approximate the results by assuming in the proof that consumers are uniformly distributed on \([w_L, w_H]\).

To say something sensible about welfare as I vary \( \lambda \), I need to explicitly normalize status utility so that status is a zero sum game. Otherwise, a change in \( \lambda \) would affect aggregate welfare just by changing the total status utility in society. I now set status utility to:
\[ U_S = \lambda \sum_{i \in N} \left( w_i - \frac{(w_L + w_H)}{2} | \alpha_i \right) \]

where the only difference is the new term \( (w_L + w_H)/2 \). With this normalization, the sum of status utility over all consumers is zero.

**Theorem 4.** Let \( m = 1 \) and \( \lambda_j = \lambda \). Say \( V(w) = \ln(w) \), \( w_H > 2w_L \) and that \( t \) is large. Then for \( \lambda \) sufficiently small, \( CV < 0 \) and the firm’s choice of advertising maximizes \( (\pi - CV) \). For \( \lambda \) sufficiently large, \( (\pi - CV) < 0 \) and the choice of advertising that maximizes \( (\pi - CV) \) is zero.

**Proof.** See appendix \( \square \)

As shown before, the firm advertises to all consumers. A quick calculation shows that the firm will sell to all consumers with wealth \( w \geq w_H/2 \), regardless of the value of \( \lambda \). It would still do so if it was forced to restrict its advertising to only those consumers who buy the status good. The result says that when concerns for status are small, trade in the status good increases consumer welfare. Trade then must also increase total welfare, since firm profits are strictly positive. It is socially optimal for the firm to advertise to all consumers, including those who do not buy the status good. When concerns for status are large, trade in the status good decreases total welfare. It would be socially optimal for trade not to take place, and so for there to be no advertising.

It is not surprising that trade increases consumer welfare when concerns for status are small. If status effects were absent then trade would result in a Pareto improvement. Here some consumers end up worse off because \( \lambda > 0 \), but consumers as a whole benefit since strategies and pay-offs are continuous in \( \lambda \).

It is less intuitive why the firm’s choice of advertising is socially optimal. Say instead the firm was forced to advertise only to consumers who buy the status good, with wealth \( w \geq w_H/2 \). The signaling value of the good is reduced compared to the equilibrium outcome. That hurts higher types who buy the status good, but benefits lower types who do not. Status is a zero sum game when \( \lambda \) is constant, so restricting the firm’s advertising amounts to a redistribution of status utility from wealthier to poorer consumers. That would seem like a social improvement, at least from the perspective of equity.

The measure of total welfare \( (\pi - CV) \), however, is only concerned with efficiency. For low values of \( \lambda \), high types are willing to pay more for a marginal unit of status than low types because they have a lower marginal utility of wealth. It is efficient for the firm to broadly advertise and transfer status utility to those who are willing to pay more for it.

Trade in the status good decreases consumer welfare when status effects are large, which comes from the fact that the \( \ln \) function decreases without bounds as wealth tends to zero. Consumers who gain from trade are never willing to give up more than their initial wealth in terms of compensation. Consumers who do not buy the status good are made worse off as \( \lambda \) increases, and their
status utility decreases linearly in $\lambda$. They need increasingly large compensating transfers as $\lambda$ increases, and their positive contribution to CV is unbounded from above.

Compensating variation therefore increases without bound as $\lambda$ tends to infinity, and firm profits $\pi$ are bounded above by total consumers wealth. That means for $\lambda$ sufficiently large, trade decreases total welfare.

7 Conclusion

This paper shows that consumer status seeking can explain why firms may advertise high-end goods to poorer consumers who will not buy them. Broad, non-targeted advertising ensures that even consumers who do not buy the good can recognize it, and so appreciate it when others buy. Broad advertising makes consumption conspicuous, by allowing consumers to signal to each other through their purchases.

An interesting avenue for further research would be to look more formally at how this mechanism influences the information firms include in their ads. As discussed in Section 5, a firm might include different information depending on whether it wants consumers to purchase the good, to recognize a variety, or to recognize the good but not to distinguish between different varieties. Looking at the latter case could shed light on why firms sometimes do not advertise product specific information, something that has not received much attention in the economics literature on advertising.

Appendix

Proof of Theorem 1. Let the firm choose $s_f = (p, a)$ such that it advertises the status good to $t'$ types and $r < t'$ types buy it. Keep $t'$ fixed, and denote the lowest type to buy the status good by $j_0$. By (1), for any value of $(w_{buy} - w_{not})$, willingness to pay $v(j)$ is increasing in type. The firm therefore set a price equal to the willingness to pay of type $j_0$. Also, for any type $j$, $v(j)$ is increasing in $(w_{buy} - w_{not})$. The firm can therefore charge the highest price by letting $j_0 = t - r + 1$, so that all consumers of type $j \geq j_0$ buy the status good. It follows that $(w_{buy} - w_{not}) = (w_H - w_L)/2 > 0$.

So $v(j_0)$ is maximized by advertising to all types, $a = T$, and the firm sets price:

$$p = v(j_0) = w_{j_0} - V^{-1}[V(w_{j_0}) - u_0 - \lambda t(w_H - w_L)]$$

Revenues are $v(j_0)(t - j_0 + 1)$, and marginal revenue drops as $j_0$ decreases. The firm chooses $j_0$ by first setting it to $t$, and then decreasing it until selling to 1 more type at a lower price yields negative marginal revenue. That is, $j_0$ is the lowest type such that:

$$(t - j_0 + 1)v(j_0) \geq (t - j_0 + 2)v(j_0 - 1)$$

The firm sells to all consumers if no such type exists.

Broad sketch of proof for Theorem 2. First consider the case where $\lambda_j = 0$ for all $j$, so where status effects are absent. The firm then just faces the standard monopolist
problem of first degree price discrimination. His optimal strategy involves dividing up the market into $m$ segments, $[j_0, j_1), [j_1, j_2), \ldots, [j_{m-1}, t]$ and selling a different variety to types in each segment.

Let the firm sell variety $x_k$ to consumers on $[j_{k-1}, j_k)$. He clearly only has an incentive to advertise one variety to each segment, so each consumer’s outside option is to only buy the composite good. A consumer with wealth $w$ has willingness to pay \( v(w) = w - V^{-1}[V(w) - u_0] \) for any variety. The firm therefore set price \( p_k = v(j_{k-1}) = w_{j_{k-1}} - V^{-1}[V(w_{j_{k-1}}) - u_0] \), the willingness to pay of the lowest type who buys variety $x_k$. Willingness to pay is strictly increasing in type, so \( p_k \) is strictly increasing in $k$.

When $\lambda_j > 0$ for all $k$, the willingness to pay of type $j$ with wealth $w_j$ for some variety $x_k$ will depend on that type’s concerns for status, $\lambda_j$, whether he is informed about variety $x_k$, and what other consumers buy variety $x_k$ in equilibrium. But since $\lambda_j$ is small for all $j$, I can assume that his willingness to pay for any variety will be close to what he would pay with $\lambda_j = 0$. That is, I can assume his willingness to pay for any variety $x_k$ as close as I’d like to $w_j - V^{-1}[V(w_j) - u_0]$, regardless of the strategies of other players.

It follows that with strictly positive status effects, the firm will still divide the market up into the same $m$ segments as without status effects. He will also still ensure that each consumer’s best outside option is to only buy the composite good. Doing otherwise would not be optimal for $\lambda_j = 0$. Since willingness to pay is continuous in $\lambda_j$, it will not be optimal for $\lambda_j > 0$ but small.

The intuition is that dividing the market up a different way or advertising such that some consumer’s best outside option is to buy another variety could increase willingness to pay through status effects, but the effect would be small. It would be outweighed by the firm’s inability to price discriminate in an optimal way.

Willingness to pay \( v(j) \) is still increasing in $j$, so the firm will not advertise a variety to higher type consumers than those who actually buy it. Say it did so, by advertising variety $x_k$ to some type $j > j_k$. The firm must set $p_k$ to make type $j_{k-1}$ indifferent between buying it and only buying the composite good. That means type $j$ strictly prefers buying $x_k$ over only buying the composite good, which makes the latter no longer his best outside option.

I claim the firm advertises each variety $x_k$ to those who buy it and to all lower types, $[1, j_k)$. The argument is by induction. First consider variety $x_m$ sold to the highest types $[j_{m-1}, t]$. The firm can charge are higher price $p_m$ if all consumers recognize $x_m$, in with case buying reveals these consumers as the highest types.

Furthermore, advertising $x_m$ to all consumers does not reduce the willingness to pay of any other type $j$ for the variety $x_k$ the firm wants to sell him. If some consumer $j’$ was uninformed about $x_k$, then becoming informed about $x_m$ has two effects. Consumer $j’$ now knows that $j’ \notin [j_{m-1}, t]$ if $j$ buys variety $x_k$, but also that $j’ \notin [j_{m-1}, t]$ if $j$ take his best outside option of only buying the composite good. If consumer $j’$ was already informed about $x_k$, then becoming informed about $x_m$ only has the later effect. So in the first case the willingness to pay of consumer $j’$ does not change, and in the second case it actually increases.

Now say the firm advertises each variety $x_m, x_{m-1}, \ldots, x_{k+1}$ to all types who buy the variety and to all lower types. Consider the willingness to pay of consumers in $[j_{k-1}, j_k)$ who buy variety $x_k$. All consumers with type $[1, j_k)$ are informed about $x_m, x_{m-1}, \ldots, x_{k+1}$ sold to types $j_k$ and higher. Willingness to pay for $x_k$ increases if the firm advertises it to all consumers on $[1, j_k)$, since then each lower type recognizes that who buy $x_k$ are highest types $[1, j_k)$.

The price $p_k$ is the willingness to pay of type $j_{k-1}$ for variety $x_k$. All types $j \geq j_k$
are uninformed about variety \(x_k\). Their beliefs about type \(j_{k-1}\) do not depend on his purchasing decision, and so do not affect his willingness to pay. All types \(j \leq j_k - 1\) are informed about variety \(x_k\). If a consumer buys variety \(x_k\), they believe he is type \((j_{k-1} + j_k - 1)/2\).

If a consumer instead only buys the composite good, then consumers’ belief depend on what segment of the market they are in. Consumers who only buy the composite good in equilibrium believe that such a consumer is one of them, so with expected type \([1 + (j_0 - 1)]/2\). Consumers who buy \(x_1\) also have the same beliefs since they are informed about all varieties. Consumers who buy \(x_2\) think the consumer is either one who only buys the composite good, or one who buys variety \(x_1\). They have beliefs \([1 + (j_1 - 1)]/2\). Continuing in this way and weighing beliefs by the number of consumers in each segment implies that type \(j_{k-1}\) has the following willingness to pay for \(x_k\):

\[
p_k = w_{j_{k-1}} - V^{-1}\{V(w_{j_{k-1}}) - u_0 - \lambda_{j_{k-1}} \sum_{k' = 0}^{k} (j_{k'} - j_{k'-1}) - 1}(j_{k-1} + j_k - 1)/2 - j_{k'}\}
\]

Proof of Theorem 4. Plugging \(V(w) = \ln(w)\) into (1) gives the following willingness to pay for a consumer with wealth \(w\):

\[
v(w) = w[1 - e^{-u_0 - \frac{\lambda}{4}(w_L - w_H)}]
\]

The firm sets the price by choosing a critical type with wealth \(w_0\) such that all consumers with wealth \(w \geq w_0\) buy the status good, where \(w_0 = \max((w_H - w_0)\ln(w_H)/w_0)\). That is \(w_0 = w_H/2 > 0\), so the firm sets \(p = v(w_H/2)\) and gets profits

\[
\pi = \frac{w_H^2}{4}[1 - e^{-u_0 - \frac{\lambda}{4}(w_H - w_L)}]
\]

Consumers who buy the status good have status utility \((\lambda/2)(w_H/2 - w_L)\), and those who do not have status utility \((\lambda/4)w_H\). The definition of compensating variation for a given consumer is the transfer \(\tau\) such that:

\[
\ln(w + \tau) - \frac{\lambda}{4}w_H = \ln(w), \text{ for all } w < \frac{w_H}{2}
\]

\[
\ln(w - p + \tau) + u_0 + \frac{\lambda}{2}(w_H/2 - w_L) = \ln(w) \text{ for all } w \geq \frac{w_H}{2}
\]

Solving for \(\tau\) gives respectively:

\[
\tau = w(e^{\frac{\lambda}{4}w_H} - 1)
\]

\[
\tau = \frac{w_H}{2}[1 - e^{-u_0 - \frac{\lambda}{4}(w_H - w_L)} - w(1 - e^{-u_0 + \frac{\lambda}{4}(w_H - w_L)})]
\]

Integrating over all consumers gives the compensating variation:

\[
CV = \frac{1}{2}(e^{\frac{\lambda}{4}w_H} - 1)(\frac{w_H^2}{4} - \frac{w_L^2}{4}) + \frac{w_H^2}{4}[1 - e^{-u_0 - \frac{\lambda}{4}(w_H - w_L)}] - \frac{3w_H^2}{8}[1 - e^{-u_0 - \frac{\lambda}{4}(w_H - w_L)}]
\]

Letting \(\lambda\) tend to zero gives \(CV < 0\), and letting \(\lambda\) tend to infinity gives \(\pi - CV < 0\).

Forcing the firm to restrict its advertising amount is equivalent to a making \(\lambda\) smaller in the equations for \(CV\) and \(\pi\). Taking \(CV - \pi\) and differentiating with respect to \(\lambda\) gives the welfare effect of reducing advertising.

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\[ \frac{\partial}{\partial \lambda} (CV - \pi) = -\frac{3}{8} w_H^2 e^{-w_0} \left( \frac{w_H}{2} - w_L \right) \left( \frac{1}{2} \left( \frac{w_H}{2} - w_L \right) \right) + \frac{1}{2} \left( e^{\lambda w_H^2} - 1 \right) \left( \frac{w_H^2}{4} - w_L^2 \right) \]

Taking the limit as \( \lambda \) tends to zero leaves only the first term, with is negative. So reducing advertising would decrease total welfare.

References


