

# Imperfect Competition in a Mixed Electricity Market with Market-Based Congestion Management\*

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## Abstract

We study a model of mixed wholesale electricity market in which a profit-maximizing private generator and a public (or a regulated) generator compete to serve consumers on the same transmission network. We allow for different objective functions for the public/regulated generator, ranging from pure consumer surplus maximization to pure profit maximization. We consider a network configuration where transmission constraints may lead to network externalities. The transmission network is operated by an Independent System Operator (ISO) utilizing a market-based congestion management system. Reliability of the transmission network is achieved via explicit nodal transmission price signals to the generators. We characterize equilibria and study various issues involved, such as multiple equilibria and how to make a plausible choice among them based on (equilibrium) profits of the ISO and the public/regulated generator. We also study the impact of the public/regulated generator's objective function on equilibrium outcomes and overall welfare and characterize its optimal objective function. Our findings also have policy implications regarding the extent to which privatization of public generation assets should take place.

**Keywords:** competition on electricity networks, network externalities, mixed oligopoly

**JEL Classification:** D43, L13, L32, L94

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# 1 Introduction

The objective of this paper is to study a model of imperfectly competitive mixed wholesale electricity market in which a profit maximizing private generator and a public generator compete. We allow for different objective functions on the part of the public generator, ranging from pure consumers' surplus maximization to pure profit maximization. This makes our analysis also applicable to electricity markets where a privately owned generator is subject to regulation, and is made to follow an objective function other than profit maximization when it competes with an unregulated private generator. Alternatively, one can think of a partially privatized public generator that follows an objective function reflecting the objectives of its public and private owners in proportion to their ownership shares. In that case, the partially privatized generator may be thought of as maximizing a weighted average of the consumers' surplus (public owner's objective) and the producers' surplus (private owners' objective).

In many countries public ownership of vertically integrated franchise utilities has been the dominant structure of the electricity supply industry prior to the end of 1980's. Since then quite a few countries have witnessed a thorough restructuring in their electricity sectors. The U.K. being the prime example, a number of countries - Australia, New Zealand and Chile, to name a few - disintegrated their formerly vertically integrated franchise utilities, deregulated and opened to competition the generation and the retail segments, while keeping the natural monopoly network segments, i.e. the high-voltage transmission and distribution segments, either under public ownership or under strict regulation. Despite these developments, public ownership of generation assets and capacity is still the predominant type of ownership in many countries; and in many cases where it is not, public production is still significant in the wholesale generation segment. A policy challenge for public authorities in many countries is to what extent the public generating assets should be privatized and what the objective function of any remaining public generation companies should be.

In Europe there are quite a few countries with "mixed" wholesale electricity markets in which public generators compete with their private counterparts: EdF in France, ENEL in Italy, Statkraft in Norway, Vattenfall in Sweden, Fortum in Finland, CEZ in Czech Republic, and ESB in Ireland are all state-owned companies. EnBW in Germany is 45% owned by EdF, the French state-owned company.

The European Union's (EU) Directive 2003/54/EC aimed at ensuring a level playing field in generation markets and reducing the risk of market dominance through non-discriminatory, transparent and fairly-priced access to transmission networks. It falls short of mandating structural unbundling of the transmission component from vertically integrated utilities but it requires legal separation and independent operation of transmission systems from generation and supply interests. This directive also orders non-discriminatory and cost-reflective balancing market tariffs until there is a liquid and competitive spot market for power. Finally, Directive 2003/54/EC requires harmonization of regulatory authorities of member states and lays down a minimum set of criteria and responsibilities they should satisfy. These criteria and responsibilities can be summarized as independence from electricity industry interests and ensuring non-discrimination, effective competition and efficient functioning of the market within their respective countries.

EU's Regulation 1228/2003 of 2003, on the other hand, attempts to establish fair, cost-reflective, transparent and directly applicable rules for cross-border tariffication and calculation and allocation of available interconnection capacities. In its current configuration, and to a greater extent in the European single market scenario, a public company is going to be acting like a public firm in its own country but like a private firm in the markets of other countries. Therefore, even markets that are currently totally dominated by a public firm, like France, will become mixed oligopolies.

In fact, state-owned electricity companies, all of which are involved in electricity generation, seem to be flourishing in the new cross-national electricity markets of Europe. The Nordic electricity market, linking Norway, Sweden, and Finland is dominated by Vattenfall, Fortum/IVO and Statkraft. The most rapidly expanding multinational in Europe is EdF. EdF, a 100 percent state owned company, has activities not only in West European countries, but in Central and Eastern Europe, North America, Latin America, Asia, and Africa. In most of these markets publicly owned companies compete with privately owned companies.<sup>1</sup>

Among the top fourteen European electricity companies ordered by the size of electricity sales in 2004, six were state-owned (or the state owns a controlling majority), with EdF and ENEL being the first and the second, respectively.<sup>2</sup>

Even in the United States, known as the stronghold of the investor-owned utility model, there is a non-negligible ownership of capacity by the public authorities. As of year-end 2004 publicly owned utilities, federal power agencies and cooperatives collectively had 20.7% of the total U.S. nameplate capacity and 22.2% of the total U.S. electricity generation.<sup>3</sup> It is also worth noting that capacity owned by investor-owned utilities for the most part is under state regulation, has fixed-price obligations to native load, and is not necessarily operated for pure profit maximization purposes.

Thus, close to a quarter of the total generating capacity of the U.S. power industry is in the hands of public entities. Furthermore, it is not possible to think of the U.S. as a single market. Due to the way the industry has historically been organized, despite significant amounts of entry by independent generators and divestiture by traditional utilities, there are many small markets secluded from each other because of insufficient interregional transmission capacity and the way grid operation and pricing are set up. In some markets the share of private and public generators in production is more balanced, while in others either public or private utilities dominate.

As already mentioned above, analysis of a mixed electricity market can also be thought to apply to a wholesale power industry where regulated private firms and unregulated private firms are competing, rather than public and private firms. Thus, it would also apply to certain states in the U.S. (such as California, New York, Illinois, New Jersey, Pennsylvania, etc.), where vertically-integrated utilities that are still partially regulated compete against unregulated independent generators.

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<sup>1</sup> To cite an example, in Norway the breakdown of installed capacity by type of ownership is as follows: about 30 percent state owned, 15 percent privately owned, and the rest owned and operated by municipalities. For shares in actual production a very similar pattern is observed (we thank Tor Arnt Johnsen and Trond Espen Haug for providing us with data on Norway). See Hall (1999) for a general account of publicly owned electricity companies in Europe.

<sup>2</sup> Source: Vattenfall, Annual Report, 2005.

<sup>3</sup> Source: American Public Power Association, 2006-07 Annual Directory & Statistical Report, page 22 <<http://www.appanet.org/files/PDFs/nameplate2004.pdf>>.

In addition to the countries discussed above, many other countries, developed as well as developing, have already partly privatized and/or totally restructured their electricity industries or are in the process of doing so. It is evident that it will take a long time, if it will ever happen, for all countries to sell all of their public generation assets. Therefore, the wholesale generation segment of electricity industry may remain a mixed oligopoly for many years, with public and private firms operating together.

Profit maximization has typically not been a significant part of the objective of public generation companies. In Sweden, for example, Vattenfall's formal objective has been to break even, with depreciation on replacement values and a rate of interest on loans from the government at the bond rate level being included. Pricing in the wholesale market for bulk power has been indirectly regulated through state ownership of Vattenfall. This has established Vattenfall as a price leader, and yardstick competition between public and private generators has created a downward price pressure. As in Sweden, the formal, government-enforced regulations have historically been fairly weak in Norway and in Finland as well. Instead, the industries are to a large extent characterized by publicly owned dominant firm leadership, self-enforced club-regulation, and yardstick competition. In U.S. public generators are by law required not to maximize profit.

In this paper we model the workings of a mixed duopolistic wholesale electricity market as a two stage game. In the first stage, the public authority (government or regulator) assigns an objective function to the public (or regulated) generator. This objective function takes the form of a weighted average of consumers' surplus and profits of the public generator. In the second stage the public generator engages in a Cournot competition (i.e. simultaneous quantity-setting game) with a private generator. The private generator chooses its output to maximize its own profits, whereas the public generator makes its output decision to maximize the objective function assigned to it in the first stage, i.e. a weighted average of its own profits and consumers' surplus.

We study a three-node electricity network, which is the minimum configuration that allows us to analyze the effects of loop flows. Each pair of these nodes is connected by a transmission line with some fixed thermal capacity. The private and the public generators are located at two separate nodes and the consumers are located at the third node. There is no demand for power on nodes where the producers are located and there is no generation capacity available on the node where the consumers are located. The transmission network is subject to congestion due to capacity constraints on lines connecting the generators and consumers.

The transmission network over which the generators are connected and serve the customers is operated by an Independent System Operator (ISO) that utilizes a market-based congestion management system. Reliability of the transmission network is achieved via explicit nodal transmission price signals to the generators. Each generator is required to pay a nodal transmission congestion charge for each unit of injection and withdrawal of electricity on each node. The congestion charge on a node can be positive, zero or negative, depending on the impact of the injection (or withdrawal) on the transmission constraint. Nodal transmission prices are determined based on the principle that all participants pay proportionally for their contribution to a binding transmission line constraint the ISO has to control for. Another (and equivalent) way to look at the same principle is that all participants pay (or get paid) for the externality they cause on the other participants, in terms of exacerbating or relieving the congestion on the constrained transmission facility.

The transmission network model we employ is consistent with the congestion management and pricing methods used in most U.S. ISO and RTO (Regional Transmission Organization) markets (PJM, Midwest ISO, ISO New England, New York ISO, California ISO), called Locational Marginal Pricing or, as it is better known with its acronym, LMP. The price at each node is comprised of two separate components, energy and congestion.<sup>4</sup> The energy price is the “system lambda”, or the would-be marginal clearing price in the system in the absence of a transmission constraint. The congestion component is the charge reflecting the node’s contribution to the transmission congestion being controlled for. The total effective price on a node is the sum of the two and applies to every participant equally located at that node.

With a market for transmission congestion rights in the model, the equilibrium of the system is defined as generation levels, an energy price, and nodal transmission congestion prices such that both the energy market and the transmission congestion rights market are in equilibrium. We assume the electricity market is characterized by Cournot competition between the generators whereas the transmission congestion rights market is competitive. That is, the generators take the nodal transmission prices as given when they make their optimal short-run production decisions. The formal definition of the equilibrium concept we employ is presented in Section 3 below.

This paper is organized as follows. In Section 2 we introduce the general features of the loop flow network model we study. In Section 3 we analyze the equilibria both when the transmission constraint is not binding and when it is binding. In Section 4 we study the profits of the ISO and the public generator in each type of equilibrium and discuss how we use these for equilibrium selection when there are multiple equilibria. In Section 5 we analyze the optimal choice of objective function for the public/regulated generator as a regulatory policy. In Section 6 we provide a summary of our results and offer some concluding remarks. Proofs of all of the propositions and the results on the profits of the ISO and the public generator are relegated to the two appendices at the end.

## 2 The Model

We consider a simple model of the electricity sector where there are two generators supplying electricity to a single market.<sup>5</sup> One of these generators, denoted by  $P$ , is purely private and its objective is to maximize its profit. The other one is a public or regulated one, denote by  $R$ , and it is assumed to maximize a weighted average of consumers' surplus and its own profits. That is, the public generator's objective function is assumed to be

$$\gamma CS(\cdot) + (1 - \gamma) \Pi_R(\cdot) \tag{1}$$

where  $CS(\cdot)$  is the total consumers' surplus,  $\Pi_R(\cdot)$  is its own profit, and  $\gamma \in [0, 1]$  is the weight on consumers' surplus. Note that the case of  $\gamma = 0$  refers to a pure private generator and  $\gamma = 1$  to a generator concerned solely with maximizing consumers' surplus.

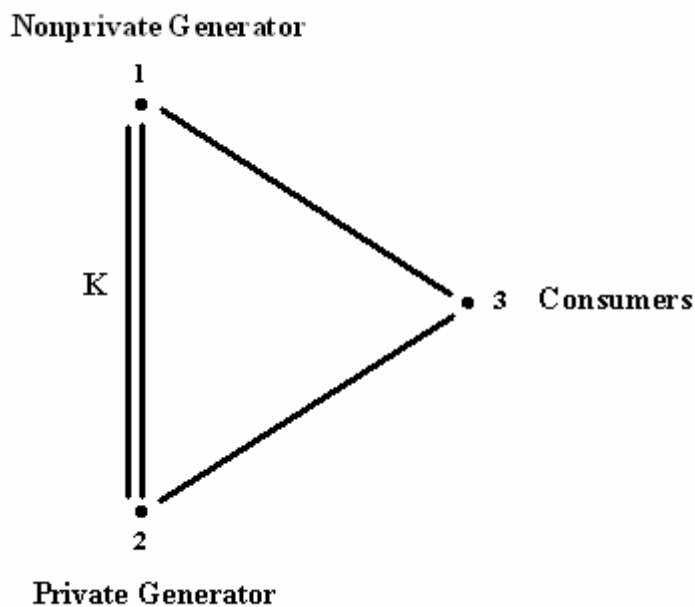
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<sup>4</sup> LMP in ISO regions consist of three components: energy, congestion and losses. However, since we will ignore transmission losses in our model, total nodal prices comprise of energy and congestion only.

<sup>5</sup> The configuration of the electricity network is similar to the model studied in Joskow and Tirole (2000).

We assume that the network is a three-node network, which is the simplest model of electricity network that involves loop-flows in electricity transmission (see Figure 1). In this case electricity sent from one node to the other not only affects the flow on the line connecting these two nodes, but also the congestion on the other two lines. We study a simple three-node network with two generation nodes and one consumption node. The public generator is located on node 1, the private generator is located on node 2, and consumers are located on node 3.<sup>6</sup> There is no generation on node 3 and no consumption on node 1 or node 2.

**Figure 1: Transmission Network Representation**



The transmission line between two generation nodes is assumed to have a given capacity of  $K$ . The implication of this in our model, which is a consequence of electricity flowing from the generators to the consumers following the path of least resistance<sup>7</sup>, is a constraint on  $q_R$  and  $q_P$  given by

$$|q_R - q_P| \leq 3K. \quad (2)$$

<sup>6</sup> There are multiple reasons for connecting the two generation nodes, even if this creates a loop flow. First, to increase the reliability of the network; in case of an outage of one of the lines connecting the generators directly to the consumers, both generators continue to supply electricity through the indirect line. In fact, a reliable operations dispatch procedure employed by all dispatchers (utility or ISO) called “n-1 contingency dispatch” is a reflection of this fact. Second, the market we are modeling can be interpreted as a sub-market in a larger interconnection with fixed available transmission capacities or transmission reservations on the lines, in which case the line connecting the two generators serves other transactions in other sub-markets and thus our ISO does not have the discretion to dismantle it.

<sup>7</sup> This is because electrons follow a unique path on an electrical transmission network determined by Kirchoff's Law rather than the direction of trade.

The grid is operated by an Independent System Operator (ISO) that is in charge of ensuring the safe and reliable utilization of the grid by auctioning transmission congestion rights, or Transmission Capacity Reservations (TCRs), as in Smeers and Wei (1997). The nodal transmission rights allow the generators to withdraw and inject up to a specific amount of electricity from and into the transmission network at a specified transmission node. As in Smeers and Wei (1997), it is assumed that transmission rights are actively traded at pre-dispatch time. Let  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  be the prices of the TCRs at nodes 1, 2 and 3, respectively.  $\lambda_i$  is the price of withdrawing a unit of electricity from node  $i$ , i.e. an entity would have to pay  $\lambda_i$  to withdraw a unit of electricity from (and pay  $-\lambda_i$  to inject into) node  $i$ , in addition to the price of the unit of electricity. Without loss of generality, we normalize the TCR prices by setting  $\lambda_1 = 0$ .

Generators compete in two markets; the electricity generation market and the TCR market. In the electricity generation market generators are assumed to engage in Cournot competition, i.e. they compete by simultaneously choosing output levels. The quantities they choose are pre-dispatch quantities submitted to the ISO that is in charge of the reliable operation of the transmission network. They also purchase and sell transmission rights at the TCR market, which is also administered by the ISO. Both generators are assumed to take TCR prices as given in making their decisions, i.e., the TCR market is competitive.

The equilibrium condition in the TCR market takes into account the externality each generator imposes on the other when transmitting a unit of electricity. When the grid is not congested, production by a generator does not impose any positive or negative externality on the other, implying TCR prices of  $\lambda_2 = \lambda_3 = 0$  in equilibrium. However, when the grid is congested, an additional unit of production by one generator creates a positive externality on the other by decongesting the line connecting them. The private generator, located on node 2, receives  $\lambda_2$  at node 2 and pays  $\lambda_3$  at node 3 for each unit of electricity it sends from node 2 to node 3. Hence the marginal transmission cost for the private generator is  $\lambda_3 - \lambda_2$ . Similarly, the public generator located at node 1 receives  $\lambda_1 (\equiv 0)$  at node 1 and pays  $\lambda_3$  at node 3 for each unit of electricity it sends from node 1 to node 3, implying that the public generator is willing to pay up to  $\lambda_3$  to benefit from the positive externality created by the additional unit of production by the private generator. In equilibrium we must have  $\lambda_3 - \lambda_2 = -\lambda_3$  (marginal cost = marginal benefit), or

$$\lambda_2 - 2\lambda_3 = 0. \quad (3)$$

When this condition is not met, a trader can earn profits through pure arbitrage, by paying both generators the market price to ramp up their productions by a unit each and collecting the associated TCRs from both. This would continue to meet the line constraint and result in a net profit for the trader if the TCR prices do not meet the above condition. In equilibrium no generator wants to hold more or fewer TCRs than it already has.

Throughout this chapter we study the short-run output decisions of the generators and assume a constant returns to scale short-run production technology of  $C_i(q_i) = c_i q_i$ , where  $q_i$  is the output of generator  $i = R, P$ .

The consumers' demand for power is represented by an affine inverse demand function,  $p(Q) = a - Q$ , where  $Q \equiv q_R + q_P$ . Defining  $\alpha_i = a - c_i$ ,  $i = R, P$ , as the *grade of efficiency* for generator  $i$ , we assume the following conditions on the demand and cost parameters:

Assumption 1:  $\alpha_i > 0$ ,  $i = R, P$ .

Assumption 2:  $2\alpha_i - \alpha_j > 0$ ,  $i, j \in \{R, P\}$ ,  $i \neq j$ .

Assumption 1 states that each generator finds it profitable to serve the whole market on its own. Assumption 2 posits that the marginal cost differential between the two generators is not "too large" and it guarantees that when both generators are pure profit maximizers, the equilibrium is an interior one when the transmission capacity constraint is not binding.

With the above cost and demand specifications, the private generator's maximization problem becomes

$$\text{Max}_{q_P \geq 0} \Pi_P = [\alpha_P - (q_R + q_P)]q_P - (\lambda_3 - \lambda_2)q_P.$$

Note that the profits of the private generator involve a separate component, namely  $(\lambda_3 - \lambda_2)q_P$ , which arises from payments due to having to acquire TCRs for each unit of electricity generated and delivered. On the other hand, the public generator's maximization problem becomes

$$\text{Max}_{q_R \geq 0} \Phi_R = \gamma \left[ \frac{1}{2} (q_R + q_P)^2 \right] + (1 - \gamma) \{ [\alpha_R - (q_R + q_P)]q_R - \lambda_3 q_R \}.$$

Note that the first term (weighted by  $\gamma$ ) involves the effect of consumers' surplus on the public generator's objective function, while the second term (weighted by  $(1 - \gamma)$ ) shows its profit arising from production and transmission.

### 3 Analysis of Equilibria

Given the above description of equilibrium in the TCR market, equilibrium in the overall system is characterized by the following conditions:

- Equilibrium in the electricity generation market:

$$\begin{aligned} \gamma(q_R + q_P) + (1 - \gamma)(\alpha_R - 2q_R - q_P) - \lambda_3 &\leq 0 \\ q_R &\geq 0 \text{ and } q_R [\gamma(q_R + q_P) + (1 - \gamma)(\alpha_R - 2q_R - q_P) - \lambda_3] &= 0 \end{aligned} \quad (4)$$

$$\begin{aligned} \alpha_P - 2q_P - q_R - \lambda_3 + \lambda_2 &\leq 0 \\ q_P &\geq 0 \text{ and } q_P [\alpha_P - 2q_P - q_R - \lambda_3 + \lambda_2] &= 0 \end{aligned} \quad (5)$$

- Feasibility in the TCR market:

$$|q_R - q_P| \leq 3K \quad (6)$$

- Equilibrium in the TCR market:

$$\lambda_2 - 2\lambda_3 = 0 \quad (7)$$



Expressions in (4) and (5) above are the first order conditions for the public and private generators, respectively, in the (constrained) maximization problem they face when they compete by choosing their quantities independently, while taking the TCR prices  $\lambda_2$  and  $\lambda_3$  as given. As we show below, for a given  $\gamma$  and  $K$ , there may exist an "unconstrained" equilibrium where the transmission capacity constraint in (6) is not binding in equilibrium, as well as "constrained" equilibria where it is binding (each equilibrium involving different  $\lambda_2$  and  $\lambda_3$ .)

### 3.1 Unconstrained Equilibria

We first look at the case where the capacity of the line connecting the two generators,  $K$ , is sufficiently large so that the grid is not congested for any value of  $\gamma$ .<sup>8</sup> As discussed in Section 2 above, in this case we have  $\lambda_2 = \lambda_3 = 0$  in equilibrium. The public generator's response function is

$$q_R(q_P; \gamma) = \begin{cases} \frac{(1-\gamma)\alpha_R - (1-2\gamma)q_P}{2-3\gamma} & \text{for } q_P \in \left[0, \frac{(1-2\gamma)\alpha_R + (2-3\gamma)c_R}{(1-\gamma)}\right) \\ a - q_P & \text{for } q_P \in \left[\frac{(1-2\gamma)\alpha_R + (2-3\gamma)c_R}{(1-\gamma)}, a\right) \\ 0 & \text{for } q_P \in [a, \infty) \end{cases} \quad (8)$$

while the private generator's response function becomes

$$q_P(q_R) = \begin{cases} \frac{\alpha_P - q_R}{2} & \text{if } q_R \in [0, \alpha_P) \\ 0 & \text{if } q_R \in [\alpha_P, \infty) \end{cases} \quad (9)$$

Note that the ("unconstrained") response function of the public generator depends on  $\gamma$ . When both  $\gamma$  and  $q_P$  are large enough, the public generator's best response is to produce just enough to complete the total output to  $a$  (at which point the market is saturated and consumers' surplus reaches its maximum value).<sup>9</sup> For  $\gamma < \frac{1}{2}$  the public generator's and the private generator's outputs are strategic substitutes while the negative slope changes beyond  $q_P = \frac{(1-2\gamma)\alpha_R + (2-3\gamma)c_R}{(1-\gamma)}$ . For  $\gamma = \frac{1}{2}$  the public generator's response function is vertical at  $q_R = \alpha_R$  until  $q_R + q_P = a$ , at which point the slope changes to -1. For  $\gamma > \frac{1}{2}$ , public and private generators' outputs become strategic

<sup>8</sup> In fact, a sufficient condition for there to be no congestion for any  $\gamma$  is  $K \geq \frac{a}{3}$ , as shown in the analysis below.

<sup>9</sup> For  $\gamma < \frac{c_R}{a + c_R}$ , there is no kink in the response function.

complements. This is due to the fact that the public generator puts a relatively larger weight on consumer surplus than on its profits when  $\gamma > \frac{1}{2}$ .

When the private generator increases its output at a given level of public generator's output, price decreases. This leads to an increase in the consumers' surplus and a decrease in the public generator's profit. The optimal response for the public generator is to increase its output until the increase in consumer surplus weighted by  $\gamma$  is just equal to the decrease in marginal profit weighted by  $1-\gamma$ . The fact that  $\gamma$ , the weight on consumer surplus, is greater than  $\frac{1}{2}$  results in an increase rather than a decrease in

public generator's output as an optimal response to an increase in private generator's output. On the other hand, a decrease in the private generator's output will lead to a decrease in the public generator's output as the optimal response for the same reason.

Let  $q_R^U(\gamma)$  and  $q_P^U(\gamma)$  be the unconstrained equilibrium output levels of the public and private generators, respectively, for a given  $\gamma$ . The (unconstrained) equilibrium output choices by the public and private generators are

$$q_R^U(\gamma) = \begin{cases} \frac{2(1-\gamma)\alpha_R - (1-2\gamma)\alpha_P}{3-4\gamma} & \text{if } 0 \leq \gamma < \underline{\gamma} \\ \frac{(1-\gamma)\alpha_R}{2-3\gamma} & \text{if } \underline{\gamma} \leq \gamma < \bar{\gamma} \\ a & \text{if } \bar{\gamma} \leq \gamma \leq 1 \end{cases} \quad (10)$$

and

$$q_P^U(\gamma) = \begin{cases} \frac{(2-3\gamma)\alpha_P - (1-\gamma)\alpha_R}{3-4\gamma} & \text{if } 0 \leq \gamma < \underline{\gamma} \\ 0 & \text{if } \underline{\gamma} \leq \gamma \leq 1 \end{cases}, \quad (11)$$

respectively, where

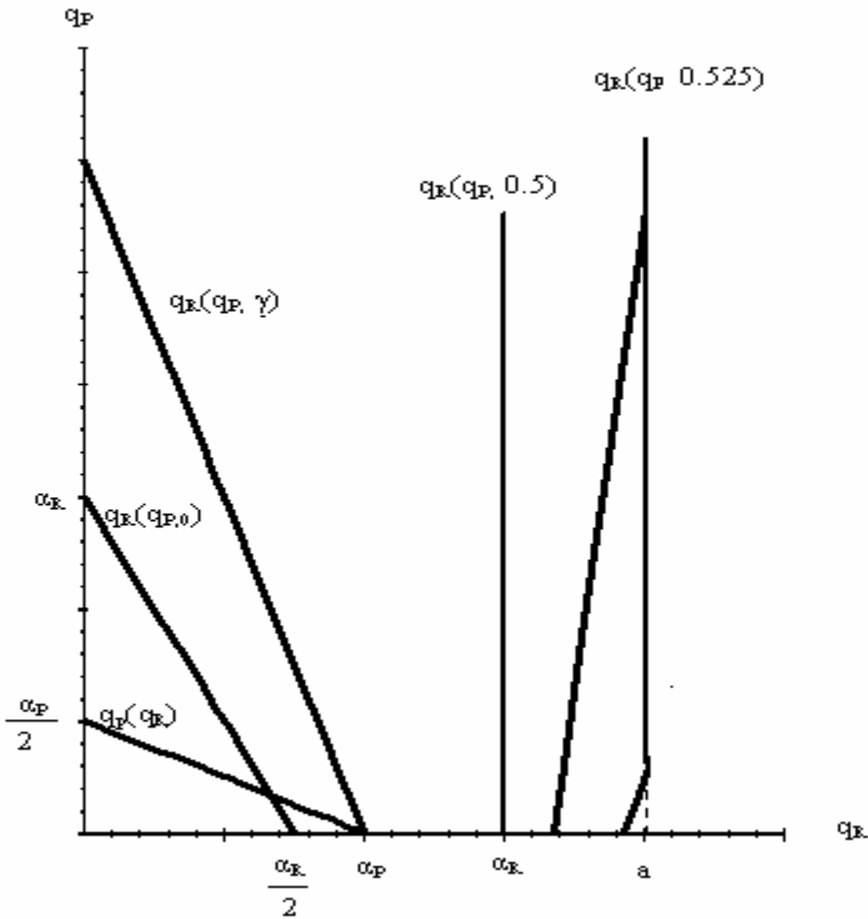
$$\underline{\gamma} = \frac{2\alpha_P - \alpha_R}{3\alpha_P - \alpha_R} \quad (12)$$

is the value of  $\gamma$  beyond which the private generator is ousted from the market and  $\bar{\gamma} = \frac{2a - \alpha_R}{3a - \alpha_R}$  is the value of  $\gamma$  beyond which the market is saturated by the public

generator's output. The total output therefore is

$$Q^U(\gamma) = \begin{cases} \frac{(1-\gamma)(\alpha_P + \alpha_R)}{3-4\gamma} & \text{if } 0 \leq \gamma < \underline{\gamma} \\ \frac{(1-\gamma)\alpha_R}{2-3\gamma} & \text{if } \underline{\gamma} \leq \gamma < \bar{\gamma} \\ a & \text{if } \bar{\gamma} \leq \gamma \leq 1 \end{cases} \quad (13)$$

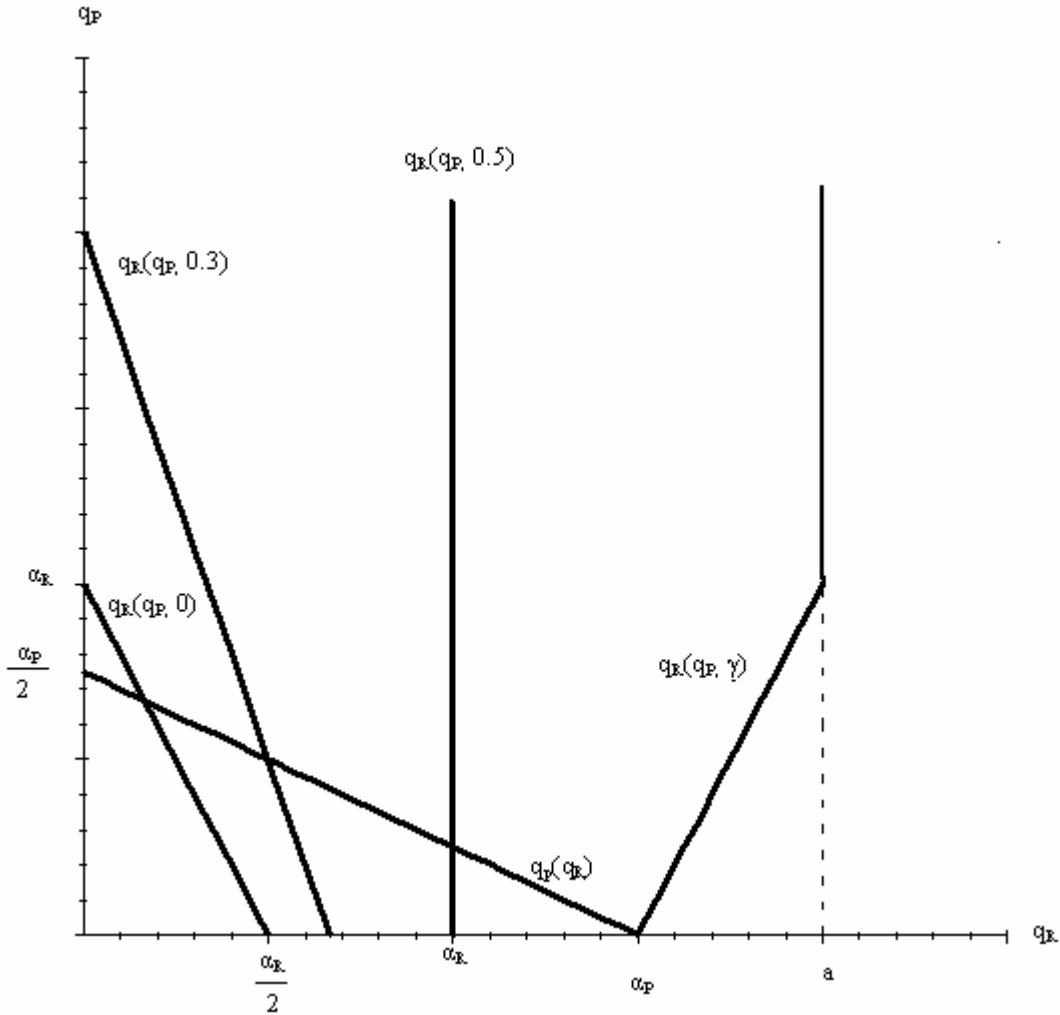
**Figure 2: Response Functions with a Less Efficient Private Generator**



Note that in Figure 2 where the public generator has a lower marginal cost, we have  $\underline{\gamma} < \frac{1}{2}$ . When the public generator has a higher marginal cost, we have  $\underline{\gamma} > \frac{1}{2}$ . That is, it takes a higher  $\gamma$  for the public generator to oust the private one from the market when the public generator is the higher cost producer.<sup>10</sup>

<sup>10</sup> Such a high level of production by the public generator may entail losses on its part.

**Figure 3: Response Functions with a More Efficient Private Generator**



Equations (10) and (11) reveal that there are two types of unconstrained equilibria. In the first case, both the public and the private generators produce a positive amount. This case corresponds to small values of  $\gamma$ , the weight on consumer's surplus in the public generator's objective function. We denote such interior unconstrained equilibria as type  $U^1$  equilibria. Whether or not the public generator is the more efficient producer impacts the characterization of  $U^1$  type equilibria. It is therefore useful to differentiate between  $U^1$  type equilibria where the public generator produces more, which we call  $U_R^1$  type, and where the private generator produces more, which we call  $U_P^1$  type. When  $\gamma$  is large enough, only the public generator produces a positive amount, and we denote such equilibria as type  $U_R^2$ .

As a direct result of Assumption 2, the public generator always produces a positive amount, even when  $\gamma = 0$ , in an unconstrained equilibrium. As shown below, the public generator's equilibrium output is increasing in  $\gamma$  in the unconstrained case, therefore,

there does not exist an unconstrained equilibrium where the public generator produces zero output.

From Equation (2) we know that whether there is congestion on the grid depends on the difference between the generators' output levels. To facilitate the characterization of the constrained equilibria, consider the difference between the unconstrained equilibrium output levels of the two generators. Letting  $\Delta q^U(\gamma) \equiv q_R^U(\gamma) - q_P^U(\gamma)$ , this difference is

$$\Delta q^U(\gamma) = \begin{cases} \alpha_R - \alpha_P + \frac{\gamma(\alpha_R + \alpha_P)}{3 - 4\gamma} & \text{if } 0 \leq \gamma < \underline{\gamma} \\ \frac{(1-\gamma)\alpha_R}{2-3\gamma} & \text{if } \underline{\gamma} \leq \gamma < \bar{\gamma} \\ a & \text{if } \bar{\gamma} \leq \gamma \leq 1 \end{cases} \quad (14)$$

To analyze the impact of  $\gamma$  on the nature of equilibria, define  $K(\gamma)$  as the capacity level that makes the unconstrained equilibrium, characterized by equations (3) and (4), just binding for a given  $\gamma$ :

$$K(\gamma) \equiv \frac{1}{3} |\Delta q^U(\gamma)| \quad (15)$$

When the public generator's objective is solely to maximize profit, i.e. when  $\gamma = 0$ , the difference between the two output levels depends only on the marginal cost differential, as long as both generators' production levels are strictly positive. It is easily shown that

$\frac{\partial \Delta q^U(\gamma)}{\partial \gamma} \geq 0$  for all  $\gamma \in [0, 1]$ , regardless of the relative efficiency of the generators. If

$\alpha_R \geq \alpha_P$ , then  $\Delta q^U(\gamma) \geq 0$  for all  $\gamma \in [0, 1]$ . If  $\alpha_R < \alpha_P$ , then  $\Delta q^U(0) < 0$  and  $\Delta q^U(1) = a > 0$ . Since  $\Delta q^U(\gamma)$  is continuous in  $\gamma$  in the relevant region, there must exist a  $\hat{\gamma} \in [0, 1]$  such that for  $\gamma \in [0, \hat{\gamma}]$ ,  $\Delta q^U(\gamma) \leq 0$  and for  $\gamma \in (\hat{\gamma}, 1]$ ,  $\Delta q^U(\gamma) > 0$ . This threshold level of  $\gamma$  is calculated as

$$\hat{\gamma} = \frac{3(\alpha_P - \alpha_R)}{5\alpha_P - 3\alpha_R}. \quad (16)$$

Note that  $\hat{\gamma}$  is always less than  $\frac{1}{2}$ . Thus, the relative magnitudes of the unconstrained equilibrium output levels depend on the relative cost efficiency of the generators and the weight attached to the consumer surplus in the public generator's objective function. If the public generator is more efficient than the private one, then the public generator produces more for any  $\gamma$ . If, on the other hand, the private generator is more efficient, then the private generator produces more for smaller levels of  $\gamma$ , i.e. for  $\gamma < \hat{\gamma}$ , while the public generator's output is higher as  $\gamma$  increases beyond  $\hat{\gamma}$ . Therefore, as the weight attached to consumers' surplus in the objective of the public generator increases beyond a threshold, the public generator produces more than its private counterpart despite its cost inefficiency.

**Figure 4: Threshold Capacity Levels with a Less Efficient Private Generator**

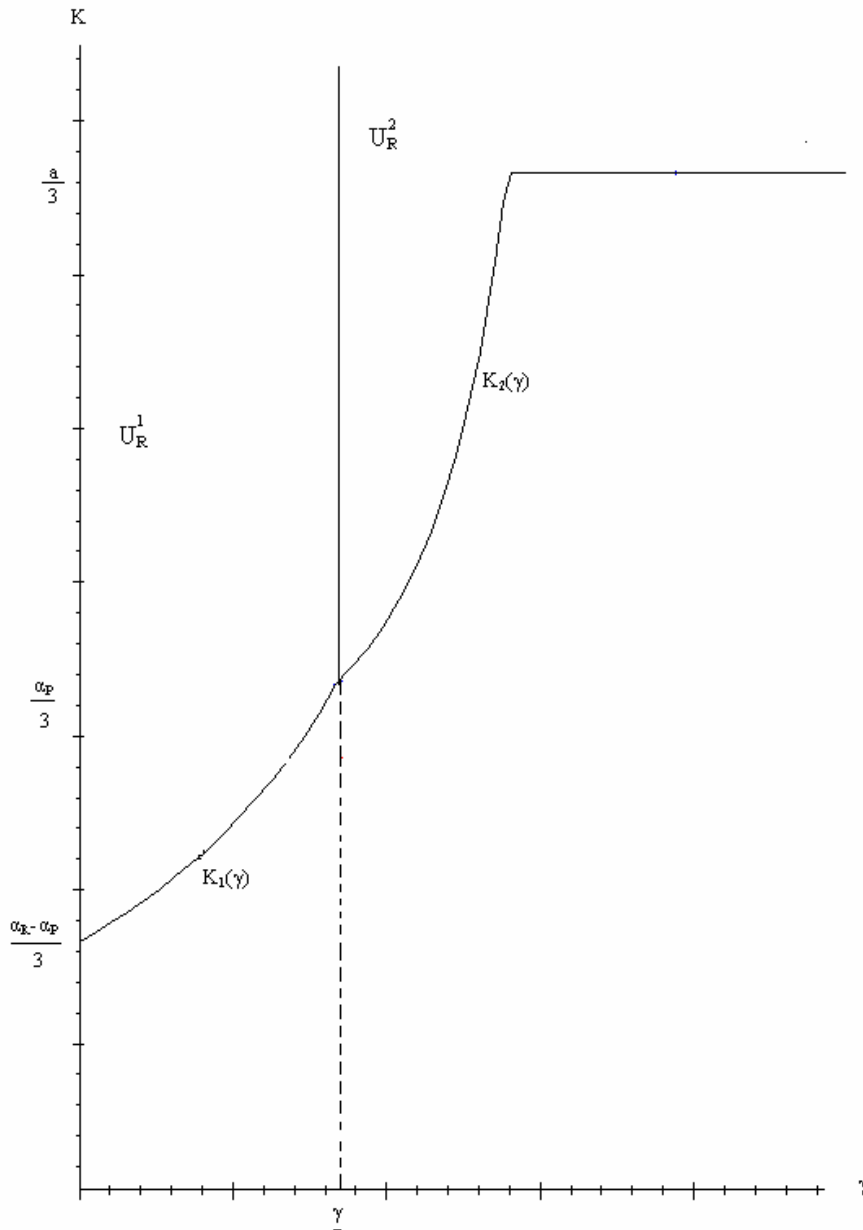


Figure 4 depicts  $K(\gamma)$  for  $\alpha_R > \alpha_P$ . Note from this figure that  $K(\gamma)$  lies within the interval  $\left[ \frac{\alpha_R - \alpha_P}{3}, \frac{a}{3} \right]$ . For a given  $\gamma \in [0, 1]$ , if  $K \geq K(\gamma)$ , then the equilibrium output levels are the unconstrained output levels characterized by Equations (3) and (4) above.

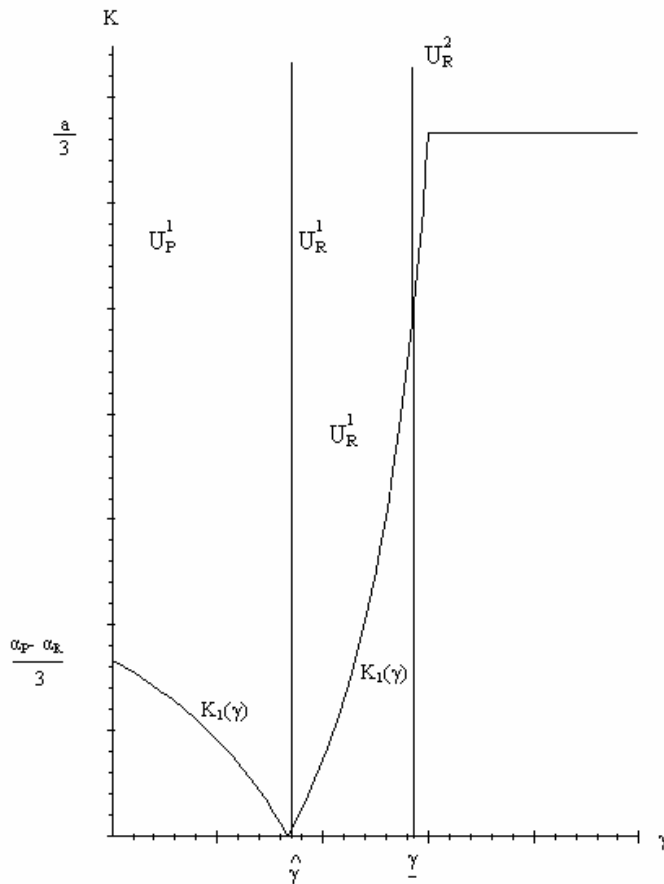
Unconstrained equilibria, of type  $U_R^1$  and  $U_R^2$ , that arise for given combinations of  $K$  and  $\gamma$  when  $\alpha_R > \alpha_P$  are depicted in Figure 4. Note that  $U_P^1$  type equilibrium does not exist when  $\alpha_R > \alpha_P$ .

Figure 5 shows  $K(\gamma)$  for  $\alpha_R < \alpha_P$ . In this case  $K(\gamma)$  resides in the interval  $[0, a/3]$ . Unconstrained equilibria for given combinations of  $K$  and  $\gamma$  are shown in Figure 5. The proposition below summarizes the results on unconstrained equilibria for both cases.

**Proposition 1:** Let  $K \geq \text{Min} \left\{ K(\gamma), \frac{a}{3} \right\}$  for a given  $\gamma$ . Then

1. If  $\alpha_R > \alpha_P$ , then the unconstrained equilibrium will be of type  $U_R^1$  for  $\gamma \in [0, \underline{\gamma}]$ , and of type  $U_R^2$  for  $\gamma \in [\underline{\gamma}, 1]$ ;
2. If  $\alpha_R < \alpha_P$ , then the unconstrained equilibrium will be of type  $U_P^1$  for  $\gamma \in [0, \hat{\gamma}]$ , of type  $U_R^1$  for  $\gamma \in [\hat{\gamma}, \underline{\gamma}]$ , and of type  $U_R^2$  for  $\gamma \in [\underline{\gamma}, 1]$ .

**Figure 5: Threshold Capacity Levels with a More Efficient Private Generator**



### 3.2 Constrained Equilibria

For a given  $\gamma$ , if  $K < K(\gamma)$ , then the equilibrium necessarily involves congestion on the grid. Since the capacity constraint involves the absolute value of the difference between output levels, there are potentially two equilibria for each level of  $K < K(\gamma)$ . That is, with congestion the TCR prices  $\lambda_2$  and  $\lambda_3$  are no longer zero, and there are two sets of  $\lambda_2$  and  $\lambda_3$  that satisfy Equations (4)-(7).

**Figure 6: Generator Response Functions for a Given  $\gamma'$**

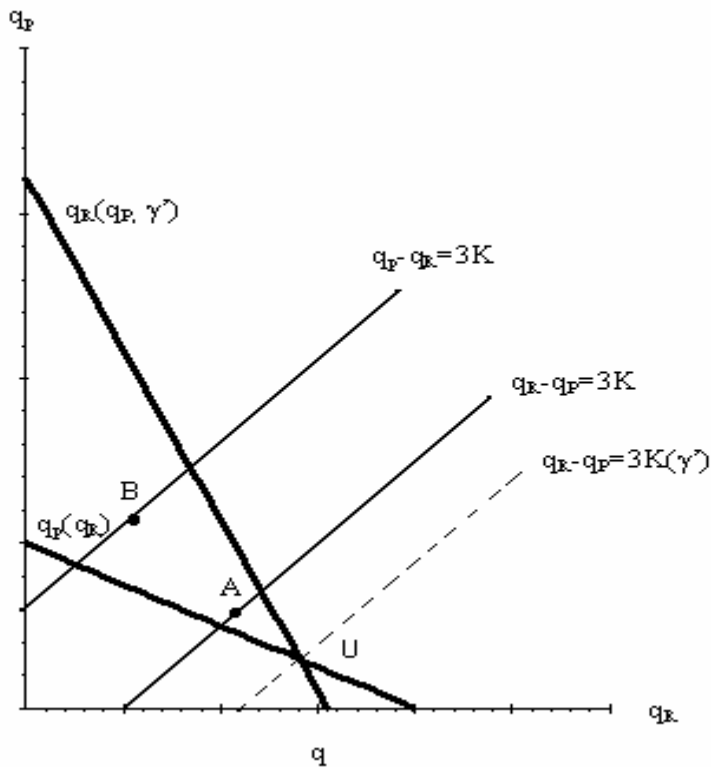


Figure 6 displays the response functions of the generators for a given  $\gamma'$ . In this case point  $U$  is an (unconstrained) equilibrium for a capacity level  $K \geq K(\gamma')$ . Take a  $K < K(\gamma')$ . The lines implied by  $|q_R - q_P| = 3K$  correspond to the capacity constraint in this case. In the TCR market nodal transmission rights are traded, and the equilibrium TCR prices shift the best response functions of the two generators such that equilibrium in the electricity market occurs at either point  $A$  or point  $B$ , where the response functions (8) and (9), the TCR market equilibrium condition  $\lambda_2 - 2\lambda_3 = 0$  and the capacity constraint lines  $|q_R - q_P| = 3K(\gamma)$  are satisfied simultaneously for  $\gamma = \gamma'$ .



## Example

We present the following simple example to illustrate the role of TCRs. Consider the case where  $\gamma = 0$  and  $\alpha_R > \alpha_P$ . In this case, the unconstrained equilibrium output levels of the public and the private generators are

$$q_R^{U^1}(0) = \frac{2\alpha_R - \alpha_P}{3} \quad (17)$$

and

$$q_P^{U^1}(0) = \frac{2\alpha_P - \alpha_R}{3}, \quad (18)$$

respectively. Since  $\alpha_R > \alpha_P$ , at the unconstrained equilibrium the public generator produces more than the private generator and  $\Delta q^{U^1}(0) = \alpha_R - \alpha_P$ . Now suppose that  $\alpha_R - \alpha_P > 3K$ , i.e. the unconstrained equilibrium is not attainable. In order to bring production in line with the capacity constraint, the effective cost of production to the public generator needs to be increased while the effective cost of production to the private generator needs to be decreased. The constrained equilibria outcomes are

$$q_R^C = \frac{\alpha_R + \alpha_P \pm 9K}{6} \quad (19)$$

$$q_P^C = \frac{\alpha_R + \alpha_P \mp 9K}{6} \quad (20)$$

$$\lambda_2^C = (\alpha_R - \alpha_P) \mp 3K > 0 \quad (21)$$

$$\lambda_3^C = \frac{(\alpha_R - \alpha_P) \mp 3K}{2} > 0 \quad (22)$$

In one of the constrained equilibria the public generator produces more, and in the other it produces less. In both cases the public generator pays  $\lambda_3^C$  (a different positive amount in each case) at the margin for each unit of electricity transmitted, thus raising its effective marginal cost to  $c_R + \lambda_3^C$ . The private generator, on the other hand, receives  $\lambda_2^C$  and pays  $\lambda_3^C$  for each unit of electricity it transmits, bringing its effective marginal cost to  $c_P + \lambda_3^C - \lambda_2^C < c_P$  in each case. With the introduction of these congestion prices, the response function of each generator moves accordingly. One could also interpret this "adjustment" in terms of prices rather than costs. The effective price the public generator receives from the sale of a unit of electricity would then be  $p^C - \lambda_3^C < p^C$ , while the private generator would be selling the same good at an effective price of  $p^C - \lambda_3^C + \lambda_2^C > p^C$ .

As in the case of unconstrained equilibria, there is a type of constrained equilibrium where both generators produce strictly positive levels of output, as well as another type of constrained equilibrium where only the public generator produces. We denote the constrained equilibria where both generators produce strictly positive amounts as equilibria of type  $C^1$ , and the equilibria where only the public generator produces as equilibria of type  $C^2$ . An equilibrium where  $q_R - q_P = 3K$  holds is denoted as  $C_R^1$ , and an

equilibrium where  $q_p - q_R = 3K$  holds is denoted as  $C_p^1$ . In other words,  $C_R^1$  is a constrained equilibrium where the public generator produces more, whereas  $C_p^1$  is a constrained equilibrium where the private generator produces more. We now turn to the characterization of each type of equilibrium.

## C<sup>1</sup> Type Equilibria

The following proposition characterizes the  $C_R^1$  type equilibrium.

**Proposition 2:** A  $C_R^1$  type equilibrium exists if and only if

$$K \leq K^{C_R^1}(\gamma) \equiv \frac{\alpha_P + (1-\gamma)\alpha_R}{9(1-\gamma)}. \quad (23)$$

The equilibrium values for the variables under consideration in this case are:

$$q_R^{C_R^1}(\gamma) = \frac{\alpha_P + (1-\gamma)\alpha_R + 3K(3-2\gamma)}{6-5\gamma} \quad (24)$$

$$q_P^{C_R^1}(\gamma) = \frac{\alpha_P + (1-\gamma)\alpha_R - 9K(1-\gamma)}{6-5\gamma} \quad (25)$$

$$\lambda_3^{C_R^1} = \frac{3(1-\gamma)\alpha_R - (3-5\gamma)\alpha_P - 3K(3-4\gamma)}{6-5\gamma} \quad (26)$$

$$\lambda_2^{C_R^1} = 2\lambda_3^{C_R^1} \quad (27)$$

**Proof:** See Appendix.

Thus,  $C_R^1$  type equilibrium exists if  $K$  is sufficiently small, i.e. when  $K < K^{C_R^1}(\gamma)$ . Note from Equation (25) that when  $K$  exceeds  $K^{C_R^1}(\gamma)$ , the private generator runs losses and hence chooses not to produce. Figure 7 and Figure 8 display the equilibria for the cases where  $\alpha_R > \alpha_P$  and  $\alpha_P > \alpha_R$ , respectively. Observe from these Figures that  $C_R^1$  type equilibrium may exist in regions where unconstrained equilibrium also exists.

**Figure 7: Types of Equilibrium with a Less Efficient Private Firm**

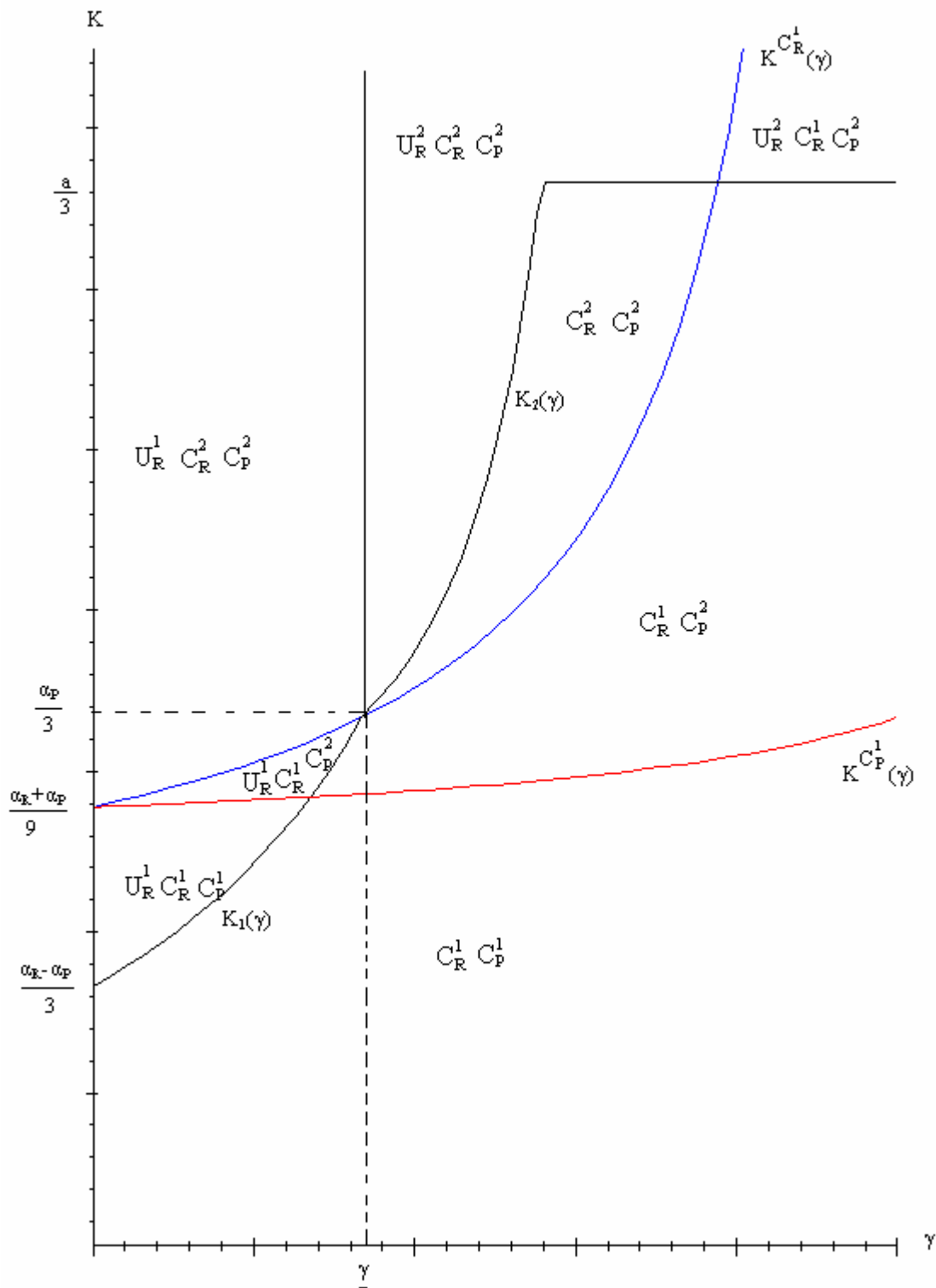
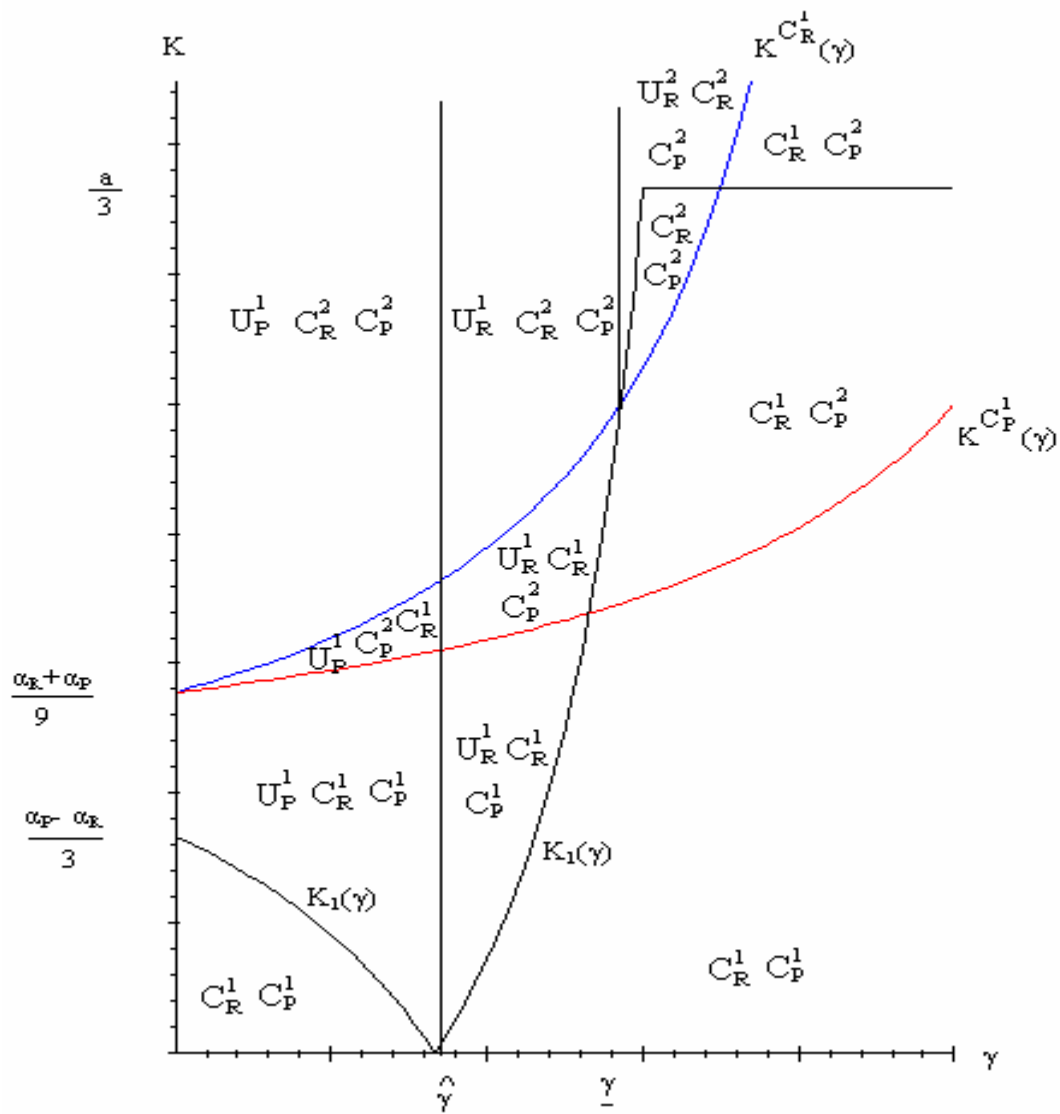


Figure 8: Types of Equilibrium with a More Efficient Private Firm



The following proposition characterizes  $C_p^1$  type equilibria.

**Proposition 3:**  $C_p^1$  type equilibria exist if and only if

$$K \leq K^{C_p^1}(\gamma) \equiv \frac{\alpha_P + (1-\gamma)\alpha_R}{3(3-2\gamma)}. \quad (28)$$

The equilibrium values for the variables under consideration in this case are:

$$q_R^{C_p^1}(\gamma) = \frac{\alpha_P + (1-\gamma)\alpha_R - 3K(3-2\gamma)}{6-5\gamma} \quad (29)$$

$$q_P^{C_p^1}(\gamma) = \frac{\alpha_P + (1-\gamma)\alpha_R + 9K(1-\gamma)}{6-5\gamma} \quad (30)$$

$$\lambda_3^{C_p^1} = \frac{3(1-\gamma)\alpha_R - (3-5\gamma)\alpha_P + 3K(3-4\gamma)}{6-5\gamma} \quad (31)$$

$$\lambda_2^{C_p^1} = 2\lambda_3^{C_p^1} \quad (32)$$

**Proof:** See Appendix.

Thus,  $C_p^1$  type equilibria exist if  $K$  is sufficiently small, this time when  $K < K^{C_p^1}(\gamma)$ . Note from Equation (24) that when  $K$  exceeds  $K^{C_p^1}(\gamma)$ , the public firm chooses not to produce. Figure 7 and Figure 8 also display  $K^{C_p^1}(\gamma)$  and show on the  $(\gamma, K)$  space the regions where  $C_p^1$  type equilibrium is obtained. Note again that  $C_p^1$  type equilibrium may exist in regions where unconstrained equilibrium also exists.

## $C^2$ Type Equilibria

$C_R^2$  type equilibria are characterized in the following proposition.

**Proposition 4:**  $C_R^2$  type equilibria exist if and only if

$$K \geq K^{C_R^2}(\gamma) \equiv \frac{\alpha_P + (1-\gamma)\alpha_R}{9(1-\gamma)}. \quad (33)$$

The equilibrium values for the variables under consideration in this case are:

$$q_R^{C_R^2}(\gamma) = 3K \quad (34)$$

$$q_P^{C_R^2}(\gamma) = 0 \quad (35)$$

$$\lambda_3^{C_R^2} = (1-\gamma)\alpha_R - 3K(2-3\gamma) \quad (36)$$

$$\lambda_2^{C_R^2} = 2\lambda_3^{C_R^2} \quad (37)$$

**Proof:** See Appendix.

Thus,  $C_R^2$  type equilibria exist in regions where  $C_p^1$  do not exist (except along the  $K^{C_p^1}(\gamma)$  curve on which they coincide). Figure 7 and Figure 8 display the regions where

$C_R^2$  type equilibria are attained. Observe again that  $C_R^2$  type equilibria may exist in regions where unconstrained equilibrium also exists.

The following proposition characterizes  $C_P^2$  type equilibria.

**Proposition 5:**  $C_P^2$  type equilibria exist if and only if

$$K \geq K^{C_P^1}(\gamma) \equiv \frac{\alpha_P + (1-\gamma)\alpha_R}{3(3-2\gamma)}. \quad (38)$$

The equilibrium values for the variables under consideration in this case are:

$$q_R^{C_P^2}(\gamma) = 0 \quad (39)$$

$$q_P^{C_P^2}(\gamma) = 3K \quad (40)$$

$$\lambda_3^{C_P^2} = 6K - \alpha_P \quad (41)$$

$$\lambda_2^{C_P^2} = 2\lambda_3^{C_P^2} \quad (42)$$

**Proof:** See Appendix.

Thus,  $C_P^2$  type equilibria exist in regions where  $C_P^1$  type equilibria do not exist (except along the  $K^{C_P^1}(\gamma)$  curve on which they coincide). Figure 7 and Figure 8 display the regions where  $C_P^2$  type equilibria are attained. Note once more that  $C_P^2$  type equilibria may exist in regions where unconstrained equilibrium also exists.

Some of our results may seem counter-intuitive at first glance; in particular, existence of an equilibrium where the private firm produces more than, or even ousts from the market, the public generator even when the public firm is the more efficient generator. It's worth noting that such counter-intuitive outcomes happen only in constrained equilibria and they are never the only possible outcomes; there are always other equilibria for the same parameter values with the "expected" outcome. However, as we discuss in detail below, in any equilibrium where the public generator is ousted from the market, the ISO runs a loss. That is, to support an equilibrium where the private generator is the only firm that produces, the ISO effectively has to "subsidize" the private generator (or a penalty to the public generator, or both) via generous transmission rights prices favoring the private generator. As we show below, once some reasonable constraints on the profits of the ISO are imposed for equilibrium selection, such counter-intuitive outcomes are eliminated.

## 4 Profits of the ISO and the Public Generator

The profits of the ISO and the public generator may be of concern for a number of reasons. The ISO is in charge of administering the TCR market and operating the transmission network. The TCR market is assumed to operate much like a competitive market, each generator taking the transmission prices it faces as given and equilibrium prices being those that equate demand and supply for transmission rights at each node. We have not ascribed a separate objective function to the ISO other than perhaps allowing it to act like a "Walrasian auctioneer" in the TCR market, announcing the final prices that will drive the TCR market into equilibrium. In our model, as in the operation of any ISO that uses market-based congestion management, TCR pricing involves

transfers to and from the ISO depending on the signs of  $\lambda_2$  and  $\lambda_3$ , as well as the relative magnitudes of  $q_R$  and  $q_P$ . The profits of the ISO are given by

$$\Pi_{ISO} = \lambda_3 q_R + (\lambda_3 - \lambda_2) q_P$$

and given that  $\lambda_2 = 2\lambda_3$  in equilibrium, the equilibrium level of ISO profits will be

$$\Pi_{ISO} = \lambda_3 (q_R - q_P). \quad (43)$$

Note that when the equilibrium is unconstrained, we have  $\lambda_2 = \lambda_3 = 0$ , and hence  $\Pi_{ISO} = 0$ . However, in the case of constrained equilibria, profits of the ISO can be positive or negative, as indicated by Equation (43) above.

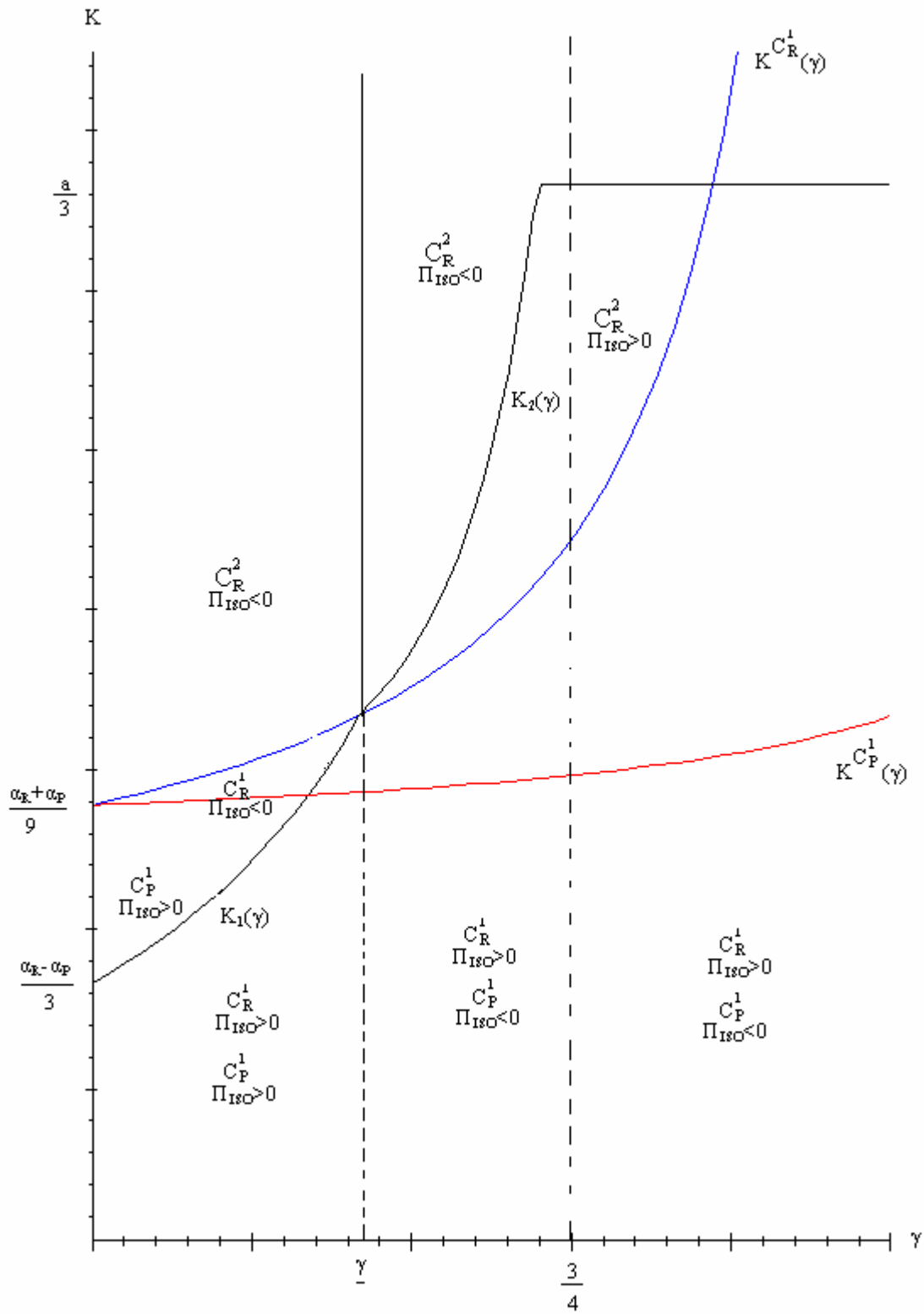
It may very well be the case that the public authority (government) requires that the ISO runs no losses. Recall also from our characterization of constrained equilibria in Section 3.2 above that for each given pair of  $K$  and  $\gamma$ , there will be one set of  $\lambda_2$  and  $\lambda_3$  that corresponds to the constraint  $q_R - q_P = 3K$  in equilibrium, and another that corresponds to  $q_P - q_R = 3K$ . It may be the case that the ISO profits are positive for one case and negative for the other. We may then use the non-negative ISO profit requirement as an equilibrium selection criterion.

In the Appendix, we present a detailed analysis of the profits of the ISO in constrained equilibria. As to be expected, the sign of the ISO profits in a constrained equilibrium (be it of  $C_R^1$ ,  $C_P^1$ ,  $C_R^2$  or  $C_P^2$  type) depends, among other things, on the relative efficiency of the public generator versus the private one. Figure 9 depicts the ISO profits in the  $(\gamma, K)$  space for the case where  $\alpha_R > \alpha_P$ , and Figure 10 does the same for the case  $\alpha_R < \alpha_P$ .

As for the profits of the public generator, recall that the public generator's objective may involve concern for consumers' surplus. In cases where a large enough weight is placed on consumers' surplus in relation to profits, i.e. for a large  $\gamma$ , a constrained equilibrium outcome may involve negative profits for the public generator. Perhaps more so than the case for ISO, it is again plausible that the public generator faces a nonnegative profit constraint. As in the case of the ISO profits, the sign of public generator's profits can also be used as an equilibrium selection criterion when there are multiple equilibria for a given pair of  $K$  and  $\gamma$ .

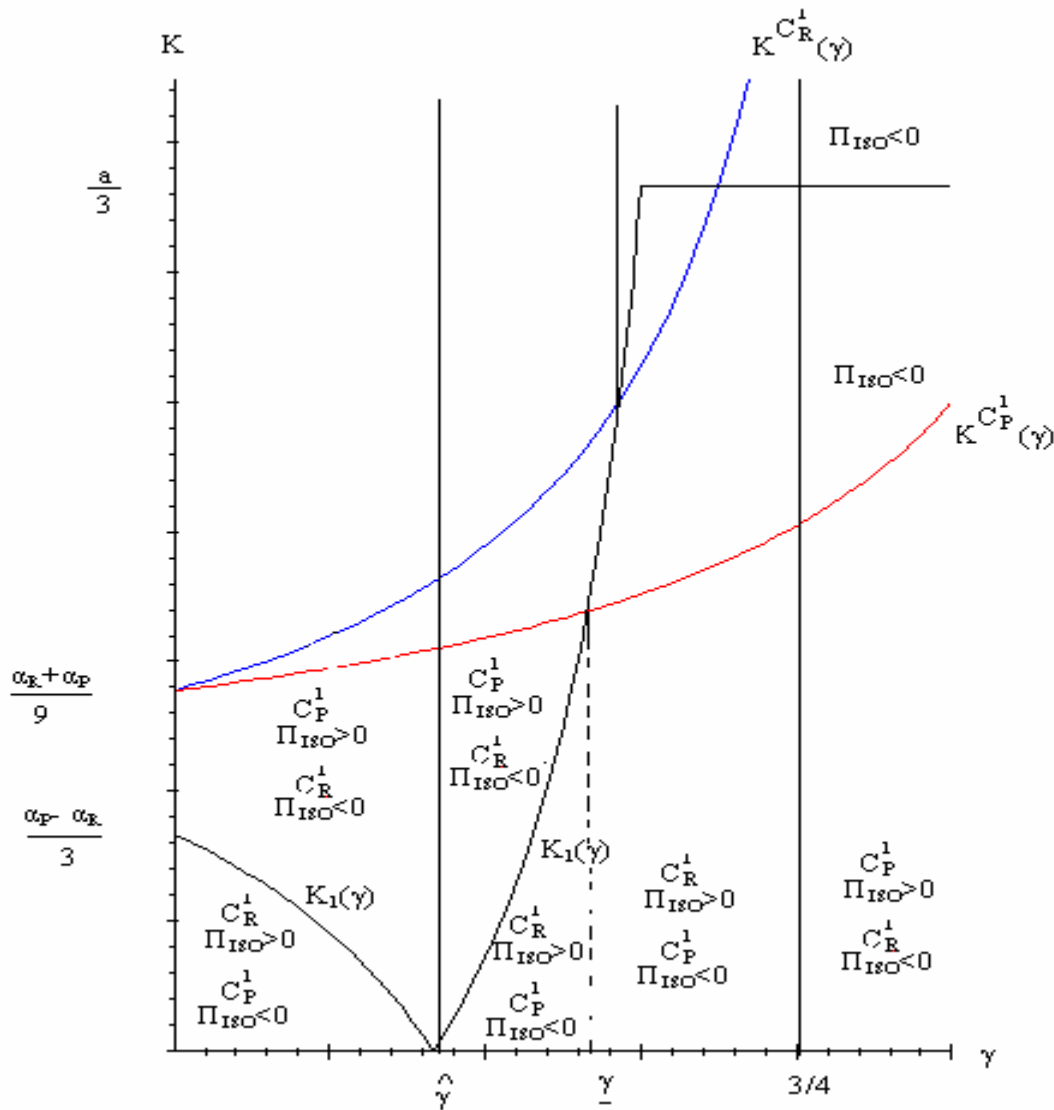
In the Appendix, we also present a detailed analysis of the profits of the public generator in all types of constrained and unconstrained equilibria, although we do not impose any constraints on the public firm's profits for equilibrium selection in the model. Unlike the case for the ISO profits, public generator's profits can be negative or positive in unconstrained equilibria as well in constrained equilibria. We note here that profits of the public generator are always nonnegative for  $U_R^1$  type equilibrium. In a  $U_R^2$  type equilibrium, where  $\gamma \in [\underline{\gamma}, \bar{\gamma}]$ , the public generator's profits are positive when  $\alpha_R > \alpha_P$ , and they are negative when  $\alpha_R < \alpha_P$ . In the region where  $\gamma \in [\bar{\gamma}, 1]$ , its profits are also strictly negative.

**Figure 9: Profits of the ISO with a Less Efficient Private Generator**





**Figure 10: Profits of the ISO with a More Efficient Private Generator**



## 5 Choice of $\gamma$ as a Regulatory Policy

The equilibrium levels of production calculated above are for a given  $K$  and  $\gamma$ . Observe that  $\gamma$ , the weight given to consumers' surplus in the public generator's objective function, can be viewed as a policy tool. This brings out the question of choosing  $\gamma$  optimally. We take the total surplus

$$W(q_R, q_P; K, \gamma) = \int_0^{q_R + q_P} P(Q) dQ - P(Q)Q + \Pi_P(q_R, q_P) + \Pi_R(q_R, q_P) + \Pi_{ISO}(q_R, q_P)$$

as the measure of welfare, which, given the specifications of our model, becomes

$$W(q_R, q_P; K, \gamma) = \alpha_R Q - \frac{Q^2}{2} - (\alpha_R - \alpha_P) q_P. \quad (44)$$

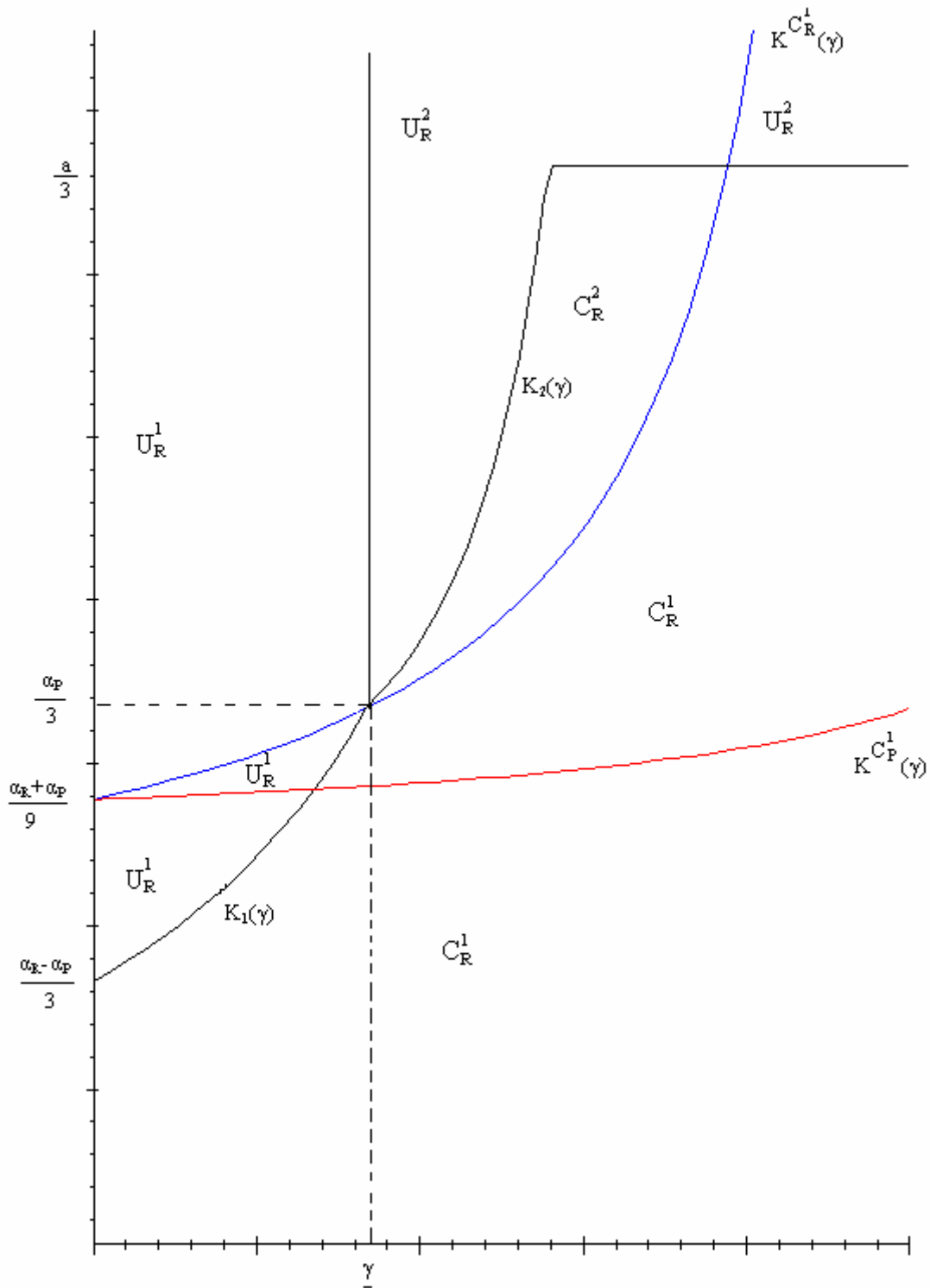
Recall that for a given  $K$  and  $\gamma$ , there will exist multiple equilibria. In order to carry out welfare analysis of any sort we have to deal with the multiplicity of equilibria. As argued in Section 4 above, a plausible criterion to select among equilibria in our case is to look at the profits of the ISO as well as those of the public generator. Using the analysis for that section (presented in the Appendix), we first eliminate equilibria that involve negative profits for the ISO. If after this elimination there still remain multiple equilibria with nonnegative profits for the ISO, we choose the one that leaves the ISO with zero profits. The ISO has a rather neutral role in the way we modeled the operation of the transmission network. In fact, the ISO simply coordinates the functioning of the TCR market. While it is plausible to insist that the ISO does not run any losses, it is also plausible to assume that it is left with minimum profits (in our case zero profits.) This approach is also consistent with the operation of major U.S. ISOs (PJM, Midwest ISO, ISO New England, New York ISO, California ISO), which are private companies that are required to operate at zero profits.

Applying these selection criteria on Figure 7 and Figure 8, we arrive at Figure 11 and Figure 12, which display the single equilibrium selected in relevant regions of the  $(\gamma, K)$  space for the cases of  $\alpha_R > \alpha_P$  and  $\alpha_R < \alpha_P$ , respectively. In computing the optimal value of  $\gamma$ , we consider the cases where the public generator is more efficient than the private generator, and where it is less efficient, separately for the sake of clarity of exposition.

### 5.1 Optimal Policy with a More Efficient Public Generator

For the case of  $\alpha_R > \alpha_P$ , the equilibrium selection process just outlined shows that when the transmission capacity  $K$  exceeds  $\frac{a}{3}$  the equilibrium is unconstrained for any  $\gamma$ , being of the  $U_R^1$  type for  $\gamma \in [0, \underline{\gamma}]$  and of the  $U_R^2$  type for  $\gamma \in [\underline{\gamma}, 1]$ . For  $\gamma \in [0, \underline{\gamma}]$  and  $K \in \left[ K_1(\gamma), \frac{a}{3} \right]$ , we have a  $U_R^1$  type equilibrium. For  $\gamma \in [0, \underline{\gamma}]$  and  $K \in [0, K_1(\gamma)]$ , we have a  $C_R^1$  type equilibrium. For  $\gamma \in [\underline{\gamma}, \bar{\gamma}]$  and  $K \in \left[ K_2(\gamma), \frac{a}{3} \right]$ , we have a type  $U_R^2$  equilibrium. For  $\gamma \in [\underline{\gamma}, \gamma^{**}]$ , where  $\gamma^{**}$  is such that  $K^{C_R^1}(\gamma^{**}) = \frac{a}{3}$ , and  $K \in \left[ K^{C_R^1}(\gamma), \text{Min} \left\{ K_2(\gamma), \frac{a}{3} \right\} \right]$  we have a  $C_R^2$  type equilibrium. Finally, for  $\gamma \in [\underline{\gamma}, 1]$  and  $K \leq \text{Min} \left\{ K^{C_R^1}(\gamma), \frac{a}{3} \right\}$  we have a  $C_R^1$  type equilibrium (see Figure 11).

**Figure 11: Equilibrium Selection with a Less Efficient Private Firm**



Define  $\gamma_2(K) \equiv (K_1)^{-1}(K)$  and  $\gamma^{C_R^1}(K) \equiv (K^{C_R^1})^{-1}(K)$  as the  $\gamma$  at which equilibrium selected changes from  $U_R^1$  to  $C_R^2$ , and from  $C_R^2$  to  $C_R^1$ , respectively. The proposition

below characterizes the optimal regulatory policy for different levels of  $K$  when the public generator is more efficient than the private one.

**Proposition 6:** Suppose  $\alpha_R > \alpha_P$ . Then the optimal regulatory policy is to set

$$\gamma^* = \begin{cases} \frac{2(\alpha_P + \alpha_R)}{\alpha_R + 5\alpha_P + 6K} & \text{if } K \in \left(0, \frac{\alpha_P + \alpha_R}{6}\right] \\ \left[\frac{6K - \alpha_R}{9K - \alpha_R}, \frac{9K - \alpha_R - \alpha_P}{9K - \alpha_R}\right] & \text{if } K \in \left[\frac{\alpha_P + \alpha_R}{6}, \frac{\alpha_R}{3}\right] \\ \frac{1}{2} & \text{if } K \in \left[\frac{\alpha_R}{3}, \infty\right) \end{cases}$$

**Proof:** See Appendix.

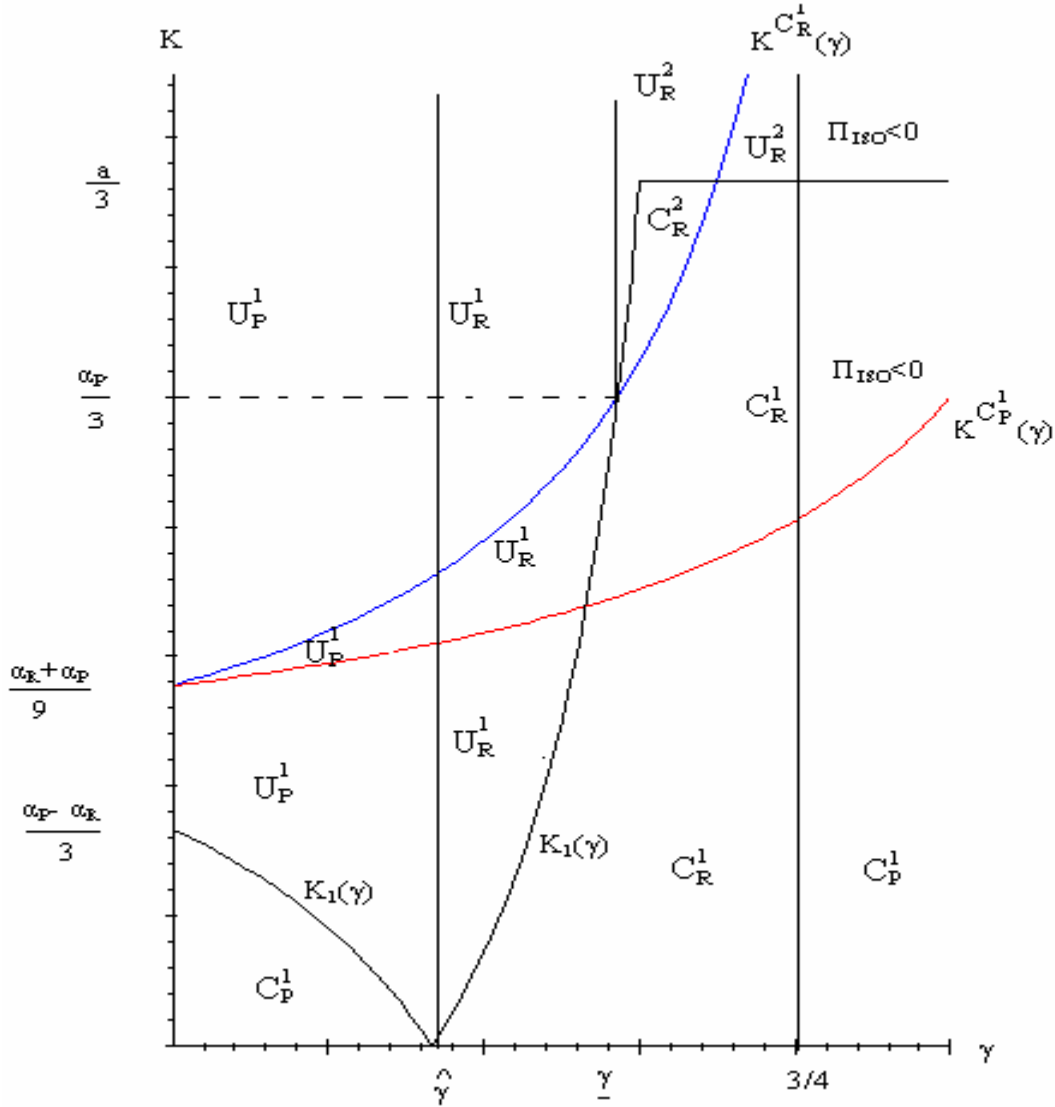
It is easy to see that the optimal  $\gamma$  in this case is never less than half. In other words, when the public generator is more efficient, the optimal regulatory policy always gives (equal or) more weight to the consumers' surplus in the public generator's objective. When the transmission line capacity is high enough, the optimal instruction is to give equal weights to profits and consumers' surplus. Weight of the consumers' surplus increases ( $\gamma$  rises from half) as the transmission line capacity falls below a threshold. However, optimal  $\gamma$  never reaches 1, that is, optimal policy stops short of instructing pure consumers' surplus maximization. Profits of the public firm are always part of optimal regulatory policy.

## 5.2 Optimal Policy with a Less Efficient Public Generator

For the case of  $\alpha_R > \alpha_P$ , the equilibrium selection process outlined above results in the following choice of equilibria in the  $(\gamma, K)$  space (see Figure 12): for  $\gamma \in [0, \hat{\gamma}]$ ,  $U_P^1$  type for  $K \geq K_1(\gamma)$  and  $C_P^1$  type for  $K < K_1(\gamma)$ ; for  $\gamma \in [\hat{\gamma}, \underline{\gamma}]$ ,  $U_R^1$  type when  $K \geq K_1(\gamma)$  and  $C_R^1$  type when  $K < K_1(\gamma)$ ; for  $\gamma \in \left[\underline{\gamma}, \frac{3}{4}\right]$ ,  $U_R^2$  type for  $K \geq \text{Min}\left\{K_2(\gamma), \frac{a}{3}\right\}$ ,  $C_R^2$  type for  $K \in \left[K^{C_R^1}(\gamma), \text{Min}\left\{K_2(\gamma), \frac{a}{3}\right\}\right]$ , and  $C_R^1$  for  $K \leq \text{Min}\left\{K^{C_R^1}(\gamma), \frac{a}{3}\right\}$ ; for  $\gamma \in \left[\frac{3}{4}, 1\right]$ ,  $U_R^2$  type for  $K \geq \frac{a}{3}$ ,  $C_R^1$  type and  $C_P^2$  type for  $K \in \left[K^{C_P^1}(\gamma), \frac{a}{3}\right]$ ,<sup>11</sup> and  $C_P^1$  type for  $K < K^{C_P^1}(\gamma)$ .

<sup>11</sup> Note that the profits of the ISO are negative for both the  $C_R^1$  and  $C_P^2$  type equilibria when  $\gamma \in [3/4, 1]$  and  $K \in K^{C_P^1}(\gamma)$ , thus our equilibrium selection criteria will not apply here. However, as will be seen in Proposition 7, the optimal  $\gamma$  never falls in this region

**Figure 12: Equilibrium Selection with a More Efficient Private Firm**



**Proposition 7:** Suppose  $\alpha_R < \alpha_P$ . Then the optimal regulatory policy is to set

$$\gamma^* = \begin{cases} \frac{3(\alpha_P - \alpha_R) - 3K}{5\alpha_P - 3\alpha_R - 4K} & \text{if } 5\alpha_R > 4\alpha_P \text{ and } K \in \left(0, \frac{6\alpha_R - 5\alpha_P}{3}\right] \\ \frac{5\alpha_R - 4\alpha_P}{7\alpha_R - 5\alpha_P} & \text{if } 5\alpha_R > 4\alpha_P \text{ and } K \in \left(\frac{6\alpha_R - 5\alpha_P}{3}, \infty\right) \\ \frac{3(\alpha_P - \alpha_R) - 3K}{5\alpha_P - 3\alpha_R - 4K} & \text{if } 5\alpha_R < 4\alpha_P \text{ and } K \in \left(0, \frac{\alpha_P - \alpha_R}{3}\right] \\ 0 & \text{if } 5\alpha_R < 4\alpha_P \text{ and } K \in \left(\frac{\alpha_P - \alpha_R}{3}, \infty\right) \end{cases}$$

**Proof:** See Appendix.

According to Proposition 7, if the private generator is more efficient, then there is a tradeoff between allocative efficiency and productive efficiency when the less efficient public generator gets to increase its output with higher  $\gamma$ . After a point, it does not pay, in terms of total surplus, to have the less efficient public generator displace production by the more efficient private generator, and the optimal objective policy stops short of maximizing consumers' surplus.

It's easily checked that  $1/2$  is the highest possible optimal choice of  $\gamma$  in this case, therefore optimal regulatory policy always puts more (or equal) weight on profits than on consumers' surplus when the private generator is more efficient. In fact, if both the transmission line capacity and the efficiency gap between the two generators (private generator being more efficient) are high enough, then the optimal regulatory policy is to instruct the public firm to maximize profits only.

## 6 Discussion and Concluding Remarks

It is well known from the Industrial Organization literature that in a standard industry structure without externalities, if a public firm competes with private firms in an imperfectly competitive market (i.e. in a 'mixed' market) social welfare might in certain cases be higher when the public firm is instructed to maximize profits instead of maximizing total surplus (consumers' surplus plus producers' surplus).<sup>12</sup> The basic intuition behind this result is that when a public firm is instructed to maximize total surplus, in some cases it tends to produce so much that gains to consumers from high consumption levels are dominated by the allocative inefficiency caused by inefficient public production displacing more efficient private production. Only in a sequential quantity setting game where the public firm is the Stackelberg leader, instructing the public firm to maximize total surplus may indeed lead to higher social welfare.<sup>13</sup>

In contrast to the studies mentioned above, in this paper we model the public generator as a firm that maximizes a weighted average of consumers' surplus and its own (short-run) profits. Consumers' surplus maximization is significantly easier to instruct or implement institutionally than total surplus maximization, making it a more realistic modeling assumption on how public/regulated firms operate. For example, in the case of a publicly-owned company, consumers' surplus can be made a part of the public firm's objective by appointing a number of consumer representatives to the board of directors or to the upper management. On the other hand, there does not seem to be an obvious way of representing the private firm's financial interests in the public firm's objective, as it may look inappropriate or create a conflict of interest to place, say, a large shareholder of the private firm in the public firm's board or upper management. In the case of a regulated private firm, like the vertically integrated utilities in the U.S. serving customers on cost of service regulated bundled rates, the state utility regulatory commission can achieve a representation of both consumers' surplus and profits of the regulated firm by allowing the regulated firm to keep a share of the profits from off-system sales. This way, the regulated firm still has an incentive to increase its profits, but not to the extent a purely private unregulated firm does, as well as an incentive to be

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<sup>12</sup> See De Fraja and Delbano (1989, 1990). See also Cremer, Marchand, and Thisse (1989) for a model in which the public firm is used as an instrument for regulating an oligopolistic market.

<sup>13</sup> See Sertel (1988).

concerned about the consumers' surplus. The bigger share of profits from off-system sales the regulated firm can keep, the larger is the weight of profits in the objective of the regulated firm is.

Also, the results on mixed markets mentioned above do not cover some of the interesting features that are observed in electricity markets. For example, when the network involves more than two nodes, e.g. when two different suppliers at two different nodes are connected to the same third node where consumption takes place, certain network externalities will arise. This is due to existence of loop flows in electricity transmission. When different nodes are connected over a transmission network, a trade between two parties can affect a non-participating third party (positively or negatively) by congesting or de-congesting the connecting lines that the third-party uses and thus altering the cost the third party faces or the quantity it will be able to sell at a particular node.

In this paper we used a competitive nodal transmission congestion rights market as the congestion management and pricing tool where each generator has to pay an explicit congestion charge in the amount of its contribution to congestion on the constrained facility. Introduction of such transmission rights brings complications, as well as benefits, to the model. The main complication we need to deal with is the multiplicity of equilibrium. For a given level of transmission capacity and a given regulatory policy, sometimes there is an equilibrium where the line is congested, as well as an equilibrium where there is no congestion on the grid. In other instances, for the same model parameters, there is an equilibrium where the line is congested in one direction, as well as an equilibrium where the line is congested in the other direction. This complication is primarily due to the flexibility that the transmission rights prices bring into the model, despite the fact that these prices cannot be set arbitrarily but they have to be such that marginal transmission price a generator faces equals to the marginal foregone profits in the electricity market. Even with this rationality restriction on them, different sets of transmission rights prices can be used to support multiple equilibrium outcomes in the electricity market and the corresponding congestion patterns.

To make any kind of a welfare analysis and choose optimal regulatory policy, the multiplicity of equilibrium issue has to be resolved. We use the profits of the ISO as the criterion for equilibrium refinement. The profits of the ISO are simply the difference between what it collects and pays out in the operation of the transmission congestion rights market. We assume that the ISO operates under a breakeven constraint, that is, its payouts in the TCR market cannot exceed its revenues. If there are still multiple equilibria after imposing the breakeven condition, we used a second refinement that the ISO set prices such that it operates with zero profits. The second refinement amounts to selecting an unconstrained equilibrium when there is also a constrained equilibrium (with different production levels and TCR prices) where the ISO makes positive profits. After imposing these two criteria on the ISO's profits, there is a unique equilibrium for each given set of parameters.

We used the equilibrium quantities computed to study the optimal choice of the objective function for the public firm in competing with its private counterpart. This exercise can be seen as analysis of optimal regulatory policy in the context of a public/private mixed oligopolistic wholesale electricity market, where the public generator plays a regulatory role by its sheer existence in the market with an objective function different than profit maximization. The objective function followed by the public generator

affects the equilibrium outcomes in a non-trivial manner and the choice of the objective function for the public generator can be viewed as choice of a particular regulatory policy.

Optimal choice of objective function for the public firm can also be viewed as a search for optimal level of privatization for the public generator. If one assumes that the objective of the public owners (consumer surplus maximization) and that of the private owners (profit maximization) are represented in the objective function of the generator according to ownership shares, then the optimal objective function will indicate whether the generator should be left in public hands (and aim at maximizing consumers' surplus in competing with its private counterpart), fully privatized (hence end up maximizing profit only just as the other, private, generator), or should be partially privatized with less than 100% of shares in private hands. In that case the question will concern the optimal level of (partial) privatization for the public generator. Level of privatization can therefore be viewed as a regulatory policy tool to be set by the public authority.

Our results indicate that the optimal regulatory policy, weight of the consumers' surplus in the public generator's objective function, depends on the capacity of the transmission line, as well as the relative cost efficiencies of the two generators. If the public generator is more efficient, optimal regulatory policy never puts more weight on profits than on consumers' surplus. When transmission line capacity is sufficiently large, profits and consumers' surplus are equally weighted. As the line capacity falls below a threshold, optimal policy puts more weight on consumers' surplus. However, regardless of model parameters, including the line capacity, the optimal instruction is never to maximize only consumers' surplus. Therefore, when the public generator is more efficient, optimal regulatory policy is a strictly convex combination of profits and consumers' surplus; pure profit maximization or pure consumers' surplus maximization is never the optimal instruction for the public generator.

If, on the other hand, the private generator is more efficient, then pure profit maximization is part of the optimal policy, though only under limiting conditions. When the cost efficiency gap is sufficiently large in favor of the private generator and the transmission line capacity is above a threshold, it is indeed optimal to instruct the public generator to maximize only profits. Aside from this limiting case, optimal regulatory policy again is always a strictly convex combination of profits and consumers' surplus. Under no circumstance is pure consumers' surplus maximization the optimal instruction. In fact, when the private generator is more efficient, profits always have a larger weight than consumers' surplus in the optimal regulatory policy. Intuitively, this is because after a point marginal welfare losses from replacing low cost private generation with high cost public generation starts to outweigh marginal gains from increasing consumers' surplus.

One might wonder, in the case of a more efficient public generator, why the optimal policy is not simply to set the weight of consumers' surplus in the public generator's objective function that will induce an equilibrium where the price of electricity is equal to the public generator's (constant) marginal cost and the public generator supplies the whole market. The reason is that such a high level of production by the public generator is not feasible due to the transmission line capacity constraint without some production from the private generator. Private generator's output is required to create counter-flow on the congested transmission line; otherwise the flow on the line exceeds safe flow limits, which cannot be tolerated by the ISO that is in charge of reliable operations of the grid. In other words, there are no market prices to support that seemingly optimal



configuration simply because the physical configuration of the grid cannot support the resulting line flows. In a nutshell, this constraint is the contribution of transmission congestion to the model, and the contribution of our results to the literature on mixed oligopolies.

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# Appendix

## A. Omitted Proofs in Chapter V

### A1. Proof of Proposition 2

$C_R^1$  type equilibrium is characterized by the simultaneous solution to the following equations:

$$\begin{aligned}\gamma(q_R + q_P) + (1-\gamma)(\alpha_R - 2q_R - q_P) - \lambda_3 &= 0 \\ \alpha_P - 2q_P - q_R - \lambda_3 + \lambda_2 &= 0 \\ q_R - q_P &= 3K \\ \lambda_2 - 2\lambda_3 &= 0\end{aligned}$$

which results in the equilibrium quantities for the variables as stated. Note that we have  $q_P^{C_R^1}(\gamma) > 0$  and  $q_R^{C_R^1}(\gamma) > q_P^{C_R^1}(\gamma)$  if and only if  $K \leq K^{C_R^1}(\gamma) \equiv \frac{\alpha_P + (1-\gamma)\alpha_R}{9(1-\gamma)}$ , as stated in the proposition.

### A2. Proof of Proposition 3

$C_P^1$  type equilibrium is characterized by the simultaneous solution to the equations

$$\begin{aligned}\gamma(q_R + q_P) + (1-\gamma)(\alpha_R - 2q_R - q_P) - \lambda_3 &= 0 \\ \alpha_P - 2q_P - q_R - \lambda_3 + \lambda_2 &= 0 \\ q_P - q_R &= 3K \\ \lambda_2 - 2\lambda_3 &= 0\end{aligned}$$

which results in the equilibrium quantities for the variables as stated in the proposition. Note that we have  $q_P^{C_P^1}(\gamma) > q_R^{C_P^1}(\gamma)$  and  $q_R^{C_P^1}(\gamma) > 0$  if and only if  $K \leq K^{C_P^1}(\gamma) \equiv \frac{\alpha_P + (1-\gamma)\alpha_R}{3(3-2\gamma)}$  as stated in the proposition.

### A3. Proof of Proposition 4

$C_R^2$  type equilibrium is characterized by the simultaneous solution to the equations

$$\begin{aligned}\gamma(q_R + q_P) + (1-\gamma)(\alpha_R - 2q_R - q_P) - \lambda_3 &= 0 \\ \alpha_P - 2q_P - q_R - \lambda_3 + \lambda_2 &\leq 0 \\ q_R &= 3K \\ \lambda_2 - 2\lambda_3 &= 0\end{aligned}$$

which results in the equilibrium quantities for the variables as stated in the proposition. Note that the inequality  $\alpha_P - 2q_P - q_R - \lambda_3 + \lambda_2 \leq 0$  is satisfied if and only if  $K \geq K^{C_R^1}(\gamma)$ , as stated in the proposition.

#### A4. Proof of Proposition 5

$C_P^2$  type equilibrium is characterized by the simultaneous solution to the equations

$$\begin{aligned}\gamma(q_R + q_P) + (1-\gamma)(\alpha_R - 2q_R - q_P) - \lambda_3 &\leq 0 \\ \alpha_P - 2q_P - q_R - \lambda_3 + \lambda_2 &\leq 0 \\ q_P &= 3K \\ \lambda_2 - 2\lambda_3 &= 0\end{aligned}$$

which results in the equilibrium quantities for the variables as stated in the proposition. Note that the inequality  $\gamma(q_R + q_P) + (1-\gamma)(\alpha_R - 2q_R - q_P) - \lambda_3 \leq 0$  is satisfied if and only if  $K \geq K^{C_P^1}(\gamma)$ , as stated in the proposition.

#### A5. Proof of Proposition 6

Let  $\alpha_R > \alpha_P$ . We first show that in this case  $\gamma = \frac{1}{2}$  will be the optimal choice if the total surplus maximization is to be attained at an unconstrained equilibrium.

Note that for each  $\gamma$  we have a unique unconstrained equilibrium with the corresponding values for  $q_R$  and  $q_P$ . We perform the welfare optimization in the  $(q_N, q_P)$  space. As  $\gamma$  increases, the equilibrium moves along the reaction function of the private generator up to  $\underline{\gamma}$  (i.e. the point where the private generator ceases production). Beyond  $\underline{\gamma}$ , the equilibrium moves along  $q_P = 0$  until  $q_R = a$  (see Figure 2) Using (44) and substituting the private generator's reaction function, the total surplus can be expressed as

$$W(q_R) = \begin{cases} \alpha_R(q_R + \frac{\alpha_P - q_R}{2}) - \frac{(q_R + \frac{\alpha_P - q_R}{2})^2}{2} - (\alpha_R - \alpha_P)(\frac{\alpha_P - q_R}{2}) & \text{if } q_R \in [0, \alpha_P) \\ \alpha_R q_R - \frac{(q_R)^2}{2} & \text{if } q_R \in [\alpha_P, a) \end{cases}$$

Differentiating with respect to  $q_R$  we get

$$\frac{\partial W(q_R)}{\partial q_R} = \begin{cases} \alpha_R - \frac{1}{4}q_R - \frac{3}{4}q_P & \text{if } q_R \in [0, \alpha_P) \\ \alpha_R - q_R & \text{if } q_R \in [\alpha_P, a) \end{cases}$$

Noting that the second order condition is satisfied, observe that  $\frac{\partial W(q_R)}{\partial q_R}$  evaluated at  $\alpha_P$  is positive. Hence total surplus is maximized by setting either  $q_R = \alpha_P$  or  $q_R = \alpha_R$ . Observe that total surplus with  $q_R = \alpha_R$  exceeds that with  $q_R = \alpha_P$ . Therefore, it is optimal to choose  $\gamma$  that will induce  $q_R = \alpha_R$ , i.e. to set  $\gamma = \frac{1}{2}$ . Note that the unconstrained equilibrium induced by  $\gamma = \frac{1}{2}$  is not attainable if  $K < \frac{\alpha_R}{3}$ .

We now determine the optimal choice if the optimum surplus is to be attained at a constrained equilibrium. For a given  $K$ , if the equilibrium is to be a constrained one, then we have  $q_R - q_P = 3K$ .<sup>14</sup> Then the maximum total surplus is achieved at  $q_R^*$  and  $q_P^*$  such that the isowelfare curve, defined as the set of output levels by the private and the public generator with constant welfare, with  $q_R^*$  and  $q_P^*$  is tangent to the constraint line  $q_R - q_P = 3K$ . This is so because welfare decreases if the equilibrium moves along  $q_R - q_P = 3K$  past this point of tangency (given that  $\alpha_R > \alpha_P$  in this case). The slope of the isowelfare curve is

$$\frac{dq_R}{dq_P} = \frac{Q - \alpha_P}{\alpha_R - Q}.$$

Setting this equal to 1 (slope of the constraint line) we get  $Q = \frac{\alpha_R + \alpha_P}{2}$ , which gives us the locus of total surplus maximizing constrained equilibria for different levels of  $K$ . For  $C_R^1$  type equilibrium the value of  $\gamma$  that induces the total surplus maximizing allocation is given by the simultaneous solution to  $q_R^{C_R^1}(\gamma)$ ,  $q_P^{C_R^1}(\gamma)$  and  $Q = \frac{\alpha_R + \alpha_P}{2}$ , resulting in

$$\gamma^* = \frac{2(\alpha_R + \alpha_P)}{\alpha_R + 5\alpha_P + 6K}.$$

Thus, it is optimal to set  $\gamma$  equal to this value, provided that the equilibrium is of  $C_R^1$  type. That is the case if  $K < \frac{\alpha_R + \alpha_P}{6}$ . For  $K > \frac{\alpha_R + \alpha_P}{6}$ , there exists a  $C_R^2$  type equilibrium for an appropriate choice of  $\gamma$ . With  $\alpha_R > \alpha_P$  it is optimal to have only the public generator produce in this region where  $K$  is sufficiently high. Observe from Figure that for all  $\gamma \in \left[ \frac{6K - \alpha_R}{9K - \alpha_R}, \frac{9K - \alpha_R - \alpha_P}{9K - \alpha_R} \right]$  we have  $C_R^2$  type equilibria, all of which involve the same output levels and hence the same total surplus.<sup>15</sup> Observe also that total surplus decreases if  $Q^* = q_R^*$  increases beyond  $\alpha_R$ . Therefore, for

<sup>14</sup> Note that with  $\alpha_R > \alpha_P$ ,  $q_R - q_P = 3K$  is the only relevant constraint line.

<sup>15</sup> Note that  $\frac{6K - \alpha_R}{9K - \alpha_R} = (K_2)^{-1}(K)$  and  $\frac{9K - \alpha_R - \alpha_P}{9K - \alpha_R} = (K^{C_R^1})^{-1}(K)$ .

$K \in \left( \frac{\alpha_P + \alpha_R}{6}, \frac{\alpha_R}{3} \right)$  the optimal policy is to set  $\gamma \in \left[ \frac{6K - \alpha_R}{9K - \alpha_R}, \frac{9K - \alpha_R - \alpha_P}{9K - \alpha_R} \right]$ . For  $K \geq \frac{\alpha_R}{3}$ ,

since the unconstrained equilibrium outcome with  $\gamma^* = \frac{1}{2}$  is attainable, it is optimal to set

$\gamma$  equal to  $\frac{1}{2}$ .

## A6. Proof of Proposition 7

Following the same argument as in the proof of Proposition 6, for a sufficiently large  $K$  we observe that total surplus is maximized by setting either  $q_R = 4\alpha_R - 3\alpha_P$  or  $q_R = \alpha_R$ . Observe that  $4\alpha_R - 3\alpha_P < \alpha_P$  and  $\alpha_R < \alpha_P$ , implying that the total surplus with  $q_R = 4\alpha_R - 3\alpha_P$  exceeds that with  $q_R = \alpha_R$ . Hence it is optimal to choose the  $\gamma$  that induces  $q_R = 4\alpha_R - 3\alpha_P$  and the corresponding  $q_P = 2(\alpha_P - \alpha_R)$ . Simple calculations show that these output levels are induced by setting  $\gamma$  equal to  $\frac{5\alpha_R - 4\alpha_P}{7\alpha_R - 5\alpha_P}$ . Note that

with  $\alpha_R < \alpha_P$ ,  $\frac{5\alpha_R - 4\alpha_P}{7\alpha_R - 5\alpha_P}$  can be positive or negative. If  $5\alpha_R > 4\alpha_P$ , then  $\frac{5\alpha_R - 4\alpha_P}{7\alpha_R - 5\alpha_P} > 0$

and checking the relevant second order condition reveals that it is indeed the global maximum. If  $5\alpha_R < 4\alpha_P$  and  $7\alpha_R > 5\alpha_P$ , then  $\frac{5\alpha_R - 4\alpha_P}{7\alpha_R - 5\alpha_P} < 0$  and the total surplus is

maximized at  $\gamma = 0$ . For the case  $7\alpha_R < 5\alpha_P$ , we have  $\frac{5\alpha_R - 4\alpha_P}{7\alpha_R - 5\alpha_P} > 0$ , but in this case

$\frac{5\alpha_R - 4\alpha_P}{7\alpha_R - 5\alpha_P}$  is a global minimum and the total surplus is again maximized at  $\gamma = 0$ .

Therefore, the total surplus is maximized at  $\frac{5\alpha_R - 4\alpha_P}{7\alpha_R - 5\alpha_P}$  if  $5\alpha_R > 4\alpha_P$ , otherwise it is

maximized at  $\gamma = 0$ . Note that the unconstrained equilibrium is attainable only if  $K \geq \frac{\alpha_P}{3}$ .

We now determine the optimal choice of  $q_R$  and  $q_P$  if the optimum surplus is to be attained at a constrained equilibrium. This is the case for  $K < \frac{\alpha_P}{3}$ . We first analyze the

case where  $5\alpha_R > 4\alpha_P$ , i.e.  $\frac{5\alpha_R - 4\alpha_P}{7\alpha_R - 5\alpha_P}$  is the total surplus maximizing value of  $\gamma$  in the

unconstrained equilibrium. Observe that with  $K = \frac{6\alpha_R - 5\alpha_P}{3}$  the equilibrium induced by

$\gamma = \frac{5\alpha_R - 4\alpha_P}{7\alpha_R - 5\alpha_P}$  is just binding. Hence for  $K \in \left[ \frac{6\alpha_R - 5\alpha_P}{3}, \frac{\alpha_P}{3} \right]$  the equilibrium moves

along the response function of the private generator for  $\gamma \in \left[0, \frac{3(\alpha_R - \alpha_P) - 3K}{3\alpha_R - 5\alpha_P - 4K}\right]$  and

along the constraint line  $q_R - q_P = 3K$  for  $\gamma \in \left[\frac{3(\alpha_R - \alpha_P) - 3K}{3\alpha_R - 5\alpha_P - 4K}, 1\right]$ . As in the

unconstrained case above, total surplus is maximized at  $\gamma = \frac{5\alpha_R - 4\alpha_P}{7\alpha_R - 5\alpha_P}$ . For

$K \in \left(0, \frac{6\alpha_R - 5\alpha_P}{3}\right]$  the equilibrium moves along  $q_P - q_R = 3K$  for

$\gamma \in \left[0, \frac{3(\alpha_P - \alpha_R) - 3K}{5\alpha_P - 3\alpha_R - 4K}\right]$ , it moves along the response function of the private generator

for  $\gamma \in \left[\frac{3(\alpha_P - \alpha_R) - 3K}{5\alpha_P - 3\alpha_R - 4K}, \frac{3(\alpha_R - \alpha_P) - 3K}{3\alpha_R - 5\alpha_P - 4K}\right]$ , and it moves along  $q_R - q_P = 3K$  for

$\gamma \in \left[\frac{3(\alpha_R - \alpha_P) - 3K}{3\alpha_R - 5\alpha_P - 4K}, 1\right]$ . Since the private generator is more efficient, the optimal

outcome will involve the private generator producing more, implying that the optimal outcome will move along  $q_P - q_R = 3K$ . The total surplus maximizing output level for the private generator in this case is

$$q_P^* = \frac{\alpha_R + \alpha_P + 6K}{4}$$

and the value of  $\gamma$  that induces this output level for the private generator is

$$\frac{2(\alpha_R + \alpha_P)}{\alpha_R + 5\alpha_P - 6K}$$

Since this is greater than  $\frac{3(\alpha_P - \alpha_R) - 3K}{5\alpha_P - 3\alpha_R - 4K}$ , it is optimal to increase  $\gamma$  in the interval

$\left[0, \frac{3(\alpha_R - \alpha_P) - 3K}{3\alpha_R - 5\alpha_P - 4K}\right]$  until the point where the equilibrium becomes unconstrained, i.e., at

$\gamma = \frac{3(\alpha_R - \alpha_P) - 3K}{3\alpha_R - 5\alpha_P - 4K}$ . This is so because increasing  $\gamma$  further results in an unconstrained

equilibrium with the less efficient public generator starting to displace production by the more efficient private generator.

If  $5\alpha_R < 4\alpha_P$ , which is the case where  $\gamma = 0$  is the total surplus maximizing choice in

the unconstrained equilibrium, then with  $K = \frac{\alpha_P - \alpha_R}{3}$  the equilibrium induced by  $\gamma = 0$  is

just binding. The rest of the analysis is identical to the one in the previous case, except

for the relevant intervals of  $K$ . For  $K \in \left[\frac{\alpha_P - \alpha_R}{3}, \frac{\alpha_P}{3}\right]$  it is optimal to set  $\gamma$  equal to 0,

whereas for  $K \in \left[0, \frac{\alpha_P - \alpha_R}{3}\right]$  it is optimal to set  $\gamma$  equal to  $\frac{3(\alpha_R - \alpha_P) - 3K}{3\alpha_R - 5\alpha_P - 4K}$ , following similar arguments as above.

## B. Profits of the ISO and the Public Firm

### B1. Profits of the ISO in Constrained Equilibria

#### $C^1$ type equilibria

When the equilibrium is of  $C_R^1$  type, we have  $q_R^{C_R^1}(\gamma) > q_P^{C_R^1}(\gamma)$ , and thus  $\Pi_{ISO} \geq 0$  if and only if  $\lambda_3 \geq 0$ . From Equation (26), this will be the case if and only if

$$K \leq K_1(\gamma) \equiv \frac{1}{3} \left[ \alpha_R - \alpha_P + \frac{\gamma(\alpha_R + \alpha_P)}{3 - 4\gamma} \right].$$

Observe from Equations (14) and (15) that  $K_1(\gamma)$  is the portion of  $K(\gamma)$  for  $\gamma \in [0, \underline{\gamma}]$ .  $K_1(\gamma)$  is thus the curve separating unconstrained equilibria from the constrained equilibria in this region. Note also that the sign of  $K_1(\gamma)$  depends on the relative magnitudes of the grades of efficiency,  $\alpha_P$  and  $\alpha_N$ .

Figure 9 shows the relationship between  $K_1(\gamma)$  and  $K^{C_R^1}(\gamma)$  when  $\alpha_R > \alpha_P$ . For  $\gamma \in \left[0, \frac{3}{4}\right]$ ,  $\Pi_{ISO} \geq 0$  if and only if  $K \leq K_1(\gamma)$ , and for  $\gamma \in \left(\frac{3}{4}, 1\right]$  we have  $\Pi_{ISO} \geq 0$  if and only if  $K \geq K_1(\gamma)$ . Thus,  $\Pi_{ISO} < 0$  for  $C_R^1$  type equilibrium in the region where  $\gamma \in [0, \underline{\gamma}]$  and  $K \in (K_1(\gamma), K^{C_R^1}(\gamma))$ , and  $\Pi_{ISO} \leq 0$  elsewhere in the  $(\gamma, K)$  space.

Figure 10 depicts  $K_1(\gamma)$  and  $K^{C_R^1}(\gamma)$  when  $\alpha_R < \alpha_P$ . In this case, for  $\gamma \in [0, \hat{\gamma})$  we have  $\Pi_{ISO} < 0$  for  $C_R^1$  type equilibrium. In the region where  $\gamma \in [\hat{\gamma}, \underline{\gamma}]$  and  $K \in (K_1(\gamma), K^{C_R^1}(\gamma))$ , we again have  $\Pi_{ISO} < 0$  for  $C_R^1$  type equilibrium.  $\Pi_{ISO} \geq 0$  in the region where  $\gamma \in \left[\hat{\gamma}, \frac{3}{4}\right]$  and  $K \leq \text{Min}\{K_1(\gamma), K^{C_R^1}(\gamma)\}$ . Finally,  $\Pi_{ISO} < 0$  for  $\gamma \in \left(\frac{3}{4}, 1\right]$ .

When the equilibrium is of  $C_P^1$  type, we have  $q_P^{C_P^1}(\gamma) > q_R^{C_P^1}(\gamma)$ , and thus  $\Pi_{ISO} \geq 0$  if and only if  $\lambda_3 \leq 0$ . From Equation (31), this is the case if and only if

$$K \leq -K_1(\gamma)$$

Figure 9 shows the relationship between  $K_1(\gamma)$  and  $K^{C_p^1}(\gamma)$  when  $\alpha_R > \alpha_P$ . For  $\gamma \in \left[0, \frac{3}{4}\right]$ ,  $\Pi_{ISO} \geq 0$  if and only if  $K \geq K_1(\gamma)$ ; and for  $\gamma \in \left(\frac{3}{4}, 1\right]$  we have  $\Pi_{ISO} \geq 0$  if and only if  $K \leq K_1(\gamma)$ . Let  $\gamma^*$  be such that  $K_1(\gamma^*) = K^{C_p^1}(\gamma^*)$ . We observe that  $\gamma^* < \underline{\gamma}$  and  $\Pi_{ISO} \geq 0$  only in the region where  $\gamma \in [0, \gamma^*]$  and  $K \in (K_1(\gamma), K^{C_p^1}(\gamma)]$ .

Figure 10 depicts  $K_1(\gamma)$  and  $K^{C_p^1}(\gamma)$  when  $\alpha_R < \alpha_P$ . In this case we have  $\Pi_{ISO} < 0$  for  $C_p^1$  type equilibrium only in the region where  $\gamma \in \left[\hat{\gamma}, \frac{3}{4}\right)$  and  $K \leq \text{Min}\{K_1(\gamma), K^{C_p^1}(\gamma)\}$ . Everywhere else in the  $(\gamma, K)$  space we have  $\Pi_{ISO} \geq 0$  in a  $C_p^1$  type equilibrium.

## **C<sup>2</sup> type equilibria**

When the equilibrium is of  $C_R^2$  type, we have  $q_P^{C_R^2}(\gamma) = 0$  and  $\Pi_{ISO} \geq 0$  if and only if  $\lambda_3 \geq 0$ . From Equation (36), this will be the case if and only if

$$K \leq K_2(\gamma) \equiv \frac{(1-\gamma)\alpha_R}{3(2-3\gamma)}.$$

Note that in the region where  $K > \text{Max}\{K_2(\gamma), K^{C_R^1}(\gamma)\}$  we have  $\Pi_{ISO} < 0$ .

When the equilibrium is of  $C_P^2$  type, we have  $q_R^{C_P^2}(\gamma) = 0$  and  $\Pi_{ISO} \geq 0$  if and only if  $\lambda_3 \leq 0$ . From Equation (41), this will be the case if and only if

$$K \leq \frac{\alpha_P}{6}.$$

Since  $K^{C_p^1}(\gamma) > \frac{\alpha_P}{6}$ , we have  $\Pi_{ISO} < 0$  in  $C_P^2$  type equilibrium, whenever it exists.

## **B2. Profits of the Public Firm**

Profits of the public generator are also of concern if it faces a break-even constraint. In this section we analyze the profits of the public generator under different types of equilibrium. The profits of the public generator is given by

$$\Pi_R = (\alpha_R - \lambda_3 - Q)q_R$$

### **Unconstrained equilibria**

Note that  $\Pi_R \geq 0$  if and only if the *marginal profit*  $M\Pi_R \equiv (\alpha_R - \lambda_3 - Q) \geq 0$ . Recall that in an unconstrained equilibrium all TCR prices are zero. In  $U_R^1$  type equilibrium the marginal profit is given by



$$M\Pi_R^{U_R^1} = \frac{(2-3\gamma)\alpha_R - (1-\gamma)\alpha_P}{3-4\gamma}. \quad (45)$$

Recall that  $U_R^1$  type equilibrium exists for  $\gamma \in [0, \bar{\gamma}]$ , and from Equation (12) it can be easily checked that  $\underline{\gamma} < \frac{3}{4}$ , implying that the denominator in Equation (45) is always positive. Hence the sign of  $M\Pi_R$  is identical to the sign of the numerator in Equation (B1), which is also positive for  $\gamma \in [0, \underline{\gamma}]$ . Since this is the relevant range for  $U_R^1$  type equilibrium,  $M\Pi_R$  and thus the profits of the public generator are always nonnegative in  $U_R^1$  type equilibrium.

$U_R^2$  type equilibrium exists only for  $\gamma \geq \underline{\gamma}$ . In the region  $\gamma \in [\underline{\gamma}, \bar{\gamma}]$ , the marginal profit for the public generator is

$$M\Pi_R^{U_R^2} = \frac{(1-2\gamma)\alpha_R}{2-3\gamma} \quad (46)$$

Recall from Section 3.1 that  $\bar{\gamma} < \frac{2}{3}$  so the denominator of  $M\Pi_R^{U_R^2}$ , and hence the profit of the public generator, is nonnegative for  $\gamma \in [\underline{\gamma}, \frac{1}{2}]$  and negative for  $\gamma \in [\frac{1}{2}, \bar{\gamma}]$ . For the case  $\alpha_R < \alpha_P$ , we have  $\underline{\gamma} > \frac{1}{2}$ , the numerator of  $M\Pi_R^{U_R^2}$ , and hence the profits of the public generator are negative for  $\gamma \in [\underline{\gamma}, \bar{\gamma}]$ , i.e., the whole region where  $U_R^2$  type equilibrium exists in this case.

Note also that in  $U_R^2$  type equilibrium, in the region where  $\gamma \in [\bar{\gamma}, 1]$ , the public generator produces  $a$ , which results in zero price for the electricity sold. Hence the profits of the public generator are strictly negative in this region (except when  $\alpha_R = 0$ ).

## Constrained equilibria

### $C^1$ type equilibria

Using Equations (24), (25) and (26), we express the marginal profit for the public generator at  $C_R^1$  type equilibria as follows:

$$M\Pi_R^{C_R^1} = \frac{\alpha_R + (1-5\gamma)\alpha_P + 3K(3-5\gamma)}{6-5\gamma} \quad (47)$$

In the region where  $\gamma \in [0, \frac{3}{5}]$ ,  $M\Pi_R^{C_R^1}$  is positive if

$$K > K^{\Pi_R^1}(\gamma) \equiv \frac{\alpha_R + (1-5\gamma)\alpha_P}{3(5\gamma-3)}.$$

In the region where  $\gamma \in \left(\frac{3}{5}, 1\right]$ ,  $M\Pi_R^{C_R^1}$  is positive if  $K < K^{\Pi_R^1}(\gamma)$ . Figure 11 shows that in the region where  $\gamma \in \left[0, \frac{\alpha_R + \alpha_P}{5\alpha_P}\right]$  we have  $K > K^{\Pi_R^1}(\gamma)$  for all  $K$ , and hence the profits of the public generator are positive wherever  $C_R^1$  type equilibrium exists in this region.<sup>16</sup> In the region where  $\gamma \in \left(\frac{\alpha_R + \alpha_P}{5\alpha_P}, \frac{3}{5}\right]$ , the profits are positive for  $K \geq K^{\Pi_R^1}(\gamma)$  and negative for  $K < K^{\Pi_R^1}(\gamma)$ . For  $\gamma \in \left(\frac{3}{5}, 1\right]$ , the profits are negative for all  $K$ .

For the case of  $C_P^1$  type equilibrium, using Equations (29), (30) and (31), we express the marginal profit for the public generator in  $C_P^1$  type equilibrium as

$$M\Pi_R^{C_P^1} = \frac{\alpha_R + (1-5\gamma)\alpha_P - 3K(3-5\gamma)}{6-5\gamma}. \quad (48)$$

In the region where  $\gamma \in \left[0, \frac{3}{5}\right]$ ,  $M\Pi_R^{C_P^1}$  is positive if

$$K < K^{\Pi_R^1}(\gamma) \equiv \frac{\alpha_R + (1-5\gamma)\alpha_P}{3(5\gamma-3)}.$$

In the region where  $\gamma \in \left(\frac{3}{5}, 1\right]$ ,  $M\Pi_R^{C_P^1}$  is positive if  $K > K^{\Pi_R^1}(\gamma)$ . Figure 11 shows that in the region where  $\gamma \in \left[0, \frac{\alpha_R + \alpha_P}{5\alpha_P}\right]$ , we have  $K > K^{\Pi_R^1}(\gamma)$  for all  $K$ , and hence the profits of the public generator are negative wherever  $C_P^1$  type equilibrium exists. In the region where  $\gamma \in \left(\frac{\alpha_R + \alpha_P}{5\alpha_P}, \frac{3}{5}\right]$ , the profits are negative for  $K \geq K^{\Pi_R^1}(\gamma)$  and positive for  $K < K^{\Pi_R^1}(\gamma)$ . For  $\gamma \in \left(\frac{3}{5}, 1\right]$ , the profits are positive for all  $K$ .

## **C<sup>2</sup> type equilibria**

Using Equations (34), (35) and (36), we write the marginal profit for the public generator in  $C_R^2$  type equilibrium as

$$M\Pi_R^{C_R^2} = \gamma\alpha_R - 3K(3\gamma-1). \quad (49)$$

In the region where  $\gamma \in \left[0, \frac{1}{3}\right]$ ,  $M\Pi_R^{C_R^2}$  is positive if

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<sup>16</sup> Observe that  $\frac{\alpha_R + \alpha_P}{5\alpha_P} < \frac{3}{5}$ .

$$K > K^{\Pi_R^2}(\gamma) \equiv \frac{\gamma\alpha_R}{3(3\gamma-1)}.$$

In the region where  $\gamma \in \left[\frac{1}{3}, 1\right]$ ,  $M\Pi_R^{C_R^2}$  is positive if  $K < K^{\Pi_R^2}(\gamma)$ . Figure 12 shows that in the region where  $\gamma \in \left[0, \frac{1}{3}\right]$  we have  $K > K^{\Pi_R^2}(\gamma)$  for all  $K$ , and hence the profits of the public generator are positive wherever  $C_R^2$  type equilibrium exists. In the region where  $\gamma \in \left[\frac{1}{3}, 1\right]$ , the profits are positive for  $K \leq K^{\Pi_R^2}(\gamma)$  and negative for  $K > K^{\Pi_R^2}(\gamma)$ .