Complementary Shifts of Technology and Development

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Abstract

I show how multiple equilibria or development traps occur in a two sector partial equilibrium model given an externality among intermediate inputs. The final goods sector achieves a higher productivity by adopting more roundabout ways of production. The growing demand by the final goods sector, in turn, induces more and more firms to enter the intermediate goods sector facilitating a wider range of production services to become available. This leads to a circularity or a self-fulfilling mechanism that triggers the economy to take off. The positive externality that arises because of the interaction between the sectors is the key to my model. Although the complementarity of newly arrived technologies effect the degree of the externality the equlibrium path of the economy is determined only by the initial degree of the intermediate input variety.

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1 Introduction

Economists have generally employed dynamic general equilibrium theory to come up with reasonable mechanisms that will resemble the observed aggregate fluctuations , development gaps or any other stylized fact of the world economy. In most cases , making realistic assumptions, for instance assuming certain market irregularities, leads to multiple equilibria. Therefore non-standard features such as increasing returns to scale, incomplete or limited participation in insurance markets or non-competitive behavior[1] were previously avoided by researchers for practical purposes. However recently, the existence of indeterminacy or multiple equilibria is being used as a positive tool for explaining the fluctuations of the aggregates by a new wave of researchers. (Benhabib, Farmer 97) It is well known that the observed irregularities in markets might lead to multiple equilibria once they are incorporated into the general equilibrium context. The difficulties associated in reconciling such features, e.g. increasing returns to scale, with a competitive theory of income distribution were overcome by using two alternative ways.

[1] A lot of mechanisms underlying the market structure, production technology that will lead to some sort of positive externalities or increasing returns to scale were mentioned in the literature. However, the importance of increasing returns to scale in indeterminacy was largely downgraded by Benhabib and Nishimura (1998). They show in a two-sector constant returns to scale model that increasing returns is not necessary for indeterminacy

One approach is to assume there exists significant externalities in the production process and another one is to introduce non-competitive behavior by the profit maximizing firms who face non-convex technologies[2] (Farmer, 93). The purpose of this paper is to give an example from the growth literature that has the first feature. More specifically, I will give an example which shows how development traps can easily arise when there exists externalities due to an inherent mechanism of production relations between two different sectors of the economy. This has important implications for explaining the patterns of development among nations as well as the issue of convergence.

One of the observed regularities in the development process is that as a country becomes more developed it has access to more roundabout ways of production. That is for a given type of final good a wider range of specialized inputs become available. This induces higher productivity as more producer services (e.g. repair, maintanence, transportation, legal advise, consulting, etc.) help to increase efficiency. Once these services become available the demand for them increase as well. So, this helps the other sector to take

[2] This idea goes back to Arrow(1962) who argued learning by doing is significant in capital accumulation process. Due to the nature of learning the production technology exhibits increasing returns to scale. How non-competitive elements may account for increasing returns has been studied by Blanchard and Kiyotaki (1987) and Benhabib and Farmer (1992)

off. This type of interdependence between sectors as examined by Murphy (1989) who suggests that "introducing efficient methods of production in one industry can increase the profitability in other sectors even though itself is not profitable."

In fact, a widely observed phenomenon is that the final goods sector achieves a higher productivity by adopting more roundabout ways of production. The growing demand by the final goods sector, in turn, induces more and more firms to enter the intermediate goods sector facilitating a wider range of production services to become available. So, there exists a circularity or a self-fulfilling mechanism that triggers the economy to take off. The multiple equilibria arises due to the fact that the intermediate goods sector has to pay start-up costs and therefore is subject to increasing returns. Once a new firm pays the setup costs and enters the intermediate goods sector it generates benefits as explained above. These benefits, however, is not fully gained back by the entering firm , but rather they are appropriated by other firms as well. The extent of how much of the growing demand is expropriated by other firms depends on the degree of substituition between the intermediate inputs and labor in the final production. The more final goods producers can substitute for intermediate goods the less is the number of firms entering the intemediate goods sector. This feature of the model drives the multiple equilibria.

2 A Model without Consumption

I will present a partial-equilibrium model with two sectors to explain the interaction mechanism between the two sectors. I will give a simple description of the model first and the implications for multiple equilibria second.

The production of final consumer goods is done by competitive firms which use a composite good X and labor L and face constant returns to scale production function , $Y = F(X, L)$.

The composite good is produced according to a CES and using $i \in [0, n]$ variety of intermediate goods which is available at any moment.

$$X = \left[\int_0^n x(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} , \sigma > 1 \tag{1}$$

where $x(i)$denotes the amount of variety i .The partial elasticity between ever pair of product is equal to σ. The restriction on σ implies that there is no essential intermediate input. Although each good is useful independent of others existence a complementarity among the differentiated inputs arise as the amount of variety effects the total factor productivity. Suppose we have n variety of goods available and let K be the total quantity of all varieties to be used. Because of the symmetry it is efficient to spend the same amount on each machine so that

$$\frac{K}{n} = x(i) \tag{2}$$

which implies that

$$X = \left[\int_0^n (\frac{K}{n})^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}} = \left[n(\frac{1}{n})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = n^{\frac{1}{\sigma-1}}.K \tag{3}$$

Since $\sigma > 1$ the productivity increases with the degree of roundaboutness of production given by $n^{\frac{1}{\sigma-1}}$. This feature of the technology can be described as increasing returns due to specialization in production.(Romer 97). The fact that the productivity in final goods sector is linked with the degree variety in intermediate goods sector drives the idea of multiple equilibria to which we will come later. The producer of the final good maximizes its profits by choosing labor and intermediate inputs optimally. Its maximization problem is given by,

$$\max F\left(\left[\int_0^n x(i)^{\frac{\sigma-1}{\sigma}}di\right]^{\frac{\sigma}{\sigma-1}}, L\right) - wL - \int_0^n p(i)x(i)di$$

Here, we are interested only in the optimal choice $x(i)$ which is given by

$$F_x\frac{\sigma}{\sigma-1}\left[\int_0^n x(i)^{\frac{\sigma-1}{\sigma}}di\right]^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}x(i)^{-\frac{1}{\sigma}} - p(i) = 0$$

or,

$$x(i) = \frac{F_x^{\sigma}X}{p(i)^{\sigma}} \tag{4}$$

This equation expresses the derived demand for the i‘th variety which is taken by the producer of good‘s i as given. Each intermediate good producer has some monopoly power over its own market. Let a be the unit of labor to be used in production of each type. The marginal cost of producing each input is then wa. The producer‘s maximization problem is then given by

$$\max_{\{p(i)\}} p(i)x(i) - wax(i)$$

substituting the demand for the intermediate good by the final good producers..

$$\max_{\{p(i)\}} \frac{F_x^{\sigma} X}{p(i)^{\sigma-1}} - wa\frac{F_x^{\sigma} X}{p(i)^{\sigma}}$$

The optimal price set by the producers is then,

$$p(i) = \frac{wa}{\sigma - 1}$$

where $\frac{\sigma}{\sigma-1}$ is the *markup*. If we normalize our choice of measurement of a (labor units) such that $a = \frac{\sigma-1}{\sigma}$ then each intermediate goods producer sets the same price $p(i) = \omega$. To see how the number of varieties affects the relative price of capital let us denote the effective price of one unit of X as P. Because of symmetry we have equal division of inputs $x(i) = x$, $p(i) = p$ and

$$X = \left[\int_0^n x(i)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}} = \left[\int_0^n x^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}} = n^{\frac{\sigma}{\sigma-1}} x$$

since P is the cost of X divided by X, it can be written as

$$P = \frac{pnx}{n^{\frac{\sigma}{\sigma-1}} x} = w.n^{\frac{-1}{\sigma-1}} \tag{5}$$

Note that the effective factor price ratio $\frac{P}{w} = n^{\frac{-1}{\sigma-1}}$ decreases with n. An increase in n makes the capital intensive goods more attractive to producers. Or in other words as more and more intermediate goods become available . This is analogous to increasing returns to scale due to specialization.

To see how factor shares in the final goods sector depend on the number of varieties in the intermediate goods sector, let us define share of capital in national income as $\alpha = \frac{F_x X}{F}$. This specification is relevant because the final goods sector is perfectly competitive. Since F is constant returns to scale α can be written as a function of the effective factor price ratio: $\alpha = \alpha(\frac{P}{w}) = \alpha(\frac{1}{n^{\frac{1}{\sigma-1}}}) \equiv A(n)$. This implies if the variety of intermediate goods goes up, then ceteris paribus there is a shift towards more capital intensive goods. The function $A(n)$ is increasing in n, whenever the elasticity of substituition between labor and the composite of intermediate goods is greater than one and vice versa.

To see how the entry and exit decisions are made by the firms let us consider how profits are effected by the total number of varieties in the intermediate goods. sector. The profits to any final goods producer are:

$$\pi = (p - aw)x = p(1 - a)x = \frac{px}{\sigma}$$

Again, because of the symmetry the profits are independent of the type of intermediate goods used so are the equilibrium prices. Let Y be the final output, then

$$\alpha Y = npx \tag{6}$$

using both equations

$$\pi = \frac{\alpha Y}{\sigma n} = \frac{A(n)}{n}\frac{Y}{\sigma} \tag{7}$$

This equations states that there are three effects that determine the profits. The first one is the negative degradation effect, or in other words the splitting out effect, represented by $\frac{1}{n}$. As the number of varieties increase the profits would fall as the firms split out their resources. The second effect is the positive substituition effect, represented by $A(n)$,which states that as the roundaboutness of production increases the intermediate goods are used more intensively by the final goods producers. And finally, the third effect is the income effect which says that as the number of varieties increase the income will increase as well.$(n \rightarrow Y)$

To finalize our analysis we need to determine the number of the intermediate good producers. We assume that all firms may enter freely into the intermediate goods sector only if they pay the setup costs which we initially ignored. The free entry condition assures that the profits are equal to these setup costs.. Let us define the setup costs as : wS where S represents the amount of labor used to setup each production facility. Then n is determined by the free entry condition $\pi = wS$ or alternatively, $\frac{A(n)}{n}\frac{Y}{\sigma} = wS$.

Rewriting both expressions :$\frac{n\sigma}{A(n)} = \frac{Y}{wS}$ where $Y = wL + n\pi$ and $L = T - nS$ where T is the total labor force. Rewriting the above equations we get

$$\frac{n\sigma}{A(n)} = \frac{T}{S} \tag{8}$$

Everything else constant a larger total labor force or smaller labor requirement for setup implies a wider range of intermediate products. That is , the size of the economy determines the variety. (or , the division of labor). At the same time the final goods producers shift to more roundabout ways of production as the differentiated intermediate inputs become more available. This would, in turn, increase the size of the market for intermediate inputs. The size of the economy depends , therefore, also on the variety of the goods. (or , the division of labor). So, there exists a circularity which leads to multiple solutions to the above equation. Any solution will depend on the shape of $\frac{n}{A(n)}$ which is determined by the magnitude of elasticity of substitution between the intermediate goods, σ, and the technology of the final goods production, $F(X, L)$. Without making rather too specific assumptions on the latter, I will try to analyze two cases. Figure 1 depicts the case with Cobb-Douglas technology in final goods production. This is the case where $A(n)$ is independent of n. A unique equilibrium exists. The intermediate goods producers are internalizing the positive returns in the final goods sector which can freel substitute between factors. Figure 2 depicts the case where the substituition in the final goods sector is limited. In this case, as n increases the profits would fall and at some point will be close to zero. This will cut the link through which the intermediate producers internalize the positive effects and the number of intermediate goods sector might start to fall. In the limited substituition case for a unique solution to exist , the start-up costs must go sufficiently down overtime to avoid firms exiting the intermediate goods sector.

Insert Figure 1 Here

Insert Figure 2 Here

2.1 An Example with CES Specification:

To see how the equilibrium is dependent on the shape of $A(n)$ consider the following specification

$$F(X, L) = (X^{\frac{\varepsilon-1}{\varepsilon}} + L^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{1-\varepsilon}}.$$

Lemma 1 : With CES, if $\varepsilon \leq \sigma$, there exists a unique equilibrium.

Note that the nature of equilibrium (or equilibria) is dependent on the shape of $A(n)$ by equation (8). If $\varepsilon \leq \sigma$ the left hand-side of (8) is strictly increasing in the positive quadrant. (proof is in the appendix). In this case we have unique equilibrium. (Figure 3)

Lemma 2 : If $\varepsilon > \sigma$, there exists multiple equilibria . The lower equilibrium represents a development trap.

In this case, the left hand-side of (8) will have a minimum at n^* and it will intersect $\frac{T}{S}$ at two points.(proof is in the appendix) If an economy starts below the lower equilibrium, one can expect that the final goods producers substitute for labor intensive technology. The demand for intermediate goods is low so that very few firms enter the intermediate sector. This represents a development trap with lower stage of growth. On the other hand, if the economy starts slightly above n(1) the range of specialized is high enough so that the final goods producers use them more intensively which leads to even more entry into the intermediate goods sector. As a result productivity grows and a take-off occurs due to this self-fulfilling mechanism.The positive feedback of increasing profits through the positive externality in the intermediate sector reinforces this feature. (Figure 4)

Insert Figure 3 Here

Insert Figure 4 Here

3 Conclusion:

The intermediate goods producers face positive externalities due to a shift towards intermediate goods (capital goods) in the final goods sector. Moreover, as the size of the economy increases the start-up costs bring increasing returns. These two features of the model constitute the basic departures from a standart convex theory of production and lead to multiple equilibria. Countries that are unable to pay the initial set-up costs are trapped in a lower equilibrium.

The deficiency of these types of models in general is the unrealistic assumption that all varieties enter the production symmetrically. A more realistic assumption would be that new varieties could be substitutes as well as complements to existing varieties. The invention of such new varieties, however, might lead to scrapping or obsolescence of others. Then the effects mentioned in this paper can not easily be shown. Introducing an R&D stage in an explicit way which distinguishes between "complementary" and "substitution" R&D would be fruitful in modeling how the new blueprints get introduced.

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5 Appendix

With CES the left hand-side of (8) can be written as . $\frac{\sigma n}{A(n)} = n + n^{\frac{\sigma-\varepsilon}{1-\sigma}}$.

Proof: We know that $A(n)$ is given by $\alpha(\frac{p}{w})$ where α is the capital share. $\frac{p}{w}$ is simply equal to $\frac{F_x}{F_l} = (\frac{X}{L})^{\frac{-1}{\varepsilon}}$ and the share of capital α is equal to $\frac{pX}{pX+wL}$. Then $\frac{1}{\alpha} = 1+\frac{wL}{pX}. = 1+(\frac{L}{X})^{\frac{\varepsilon-1}{\varepsilon}}$ Remember also that $\frac{w}{p} = n^{\frac{-1}{1-\sigma}} = (\frac{X}{L})^{\frac{-1}{\sigma}}$. Since $(\frac{L}{X})^{\frac{1}{\varepsilon}} = n^{\frac{1}{1-\sigma}}$ the last term of the previous equation $(\frac{L}{X})^{\frac{\varepsilon-1}{\varepsilon}}$ can be written as $n^{\frac{1-\varepsilon}{1-\sigma}}$. Note that $\frac{\sigma n}{A(n)}$ schedule has an extreme point at $n^* = (\frac{\varepsilon-\sigma}{\sigma-1})^{\frac{\sigma-1}{\varepsilon-1}}$ which leaves us with two cases. $\varepsilon \leq \sigma$ and $\varepsilon > \sigma$.

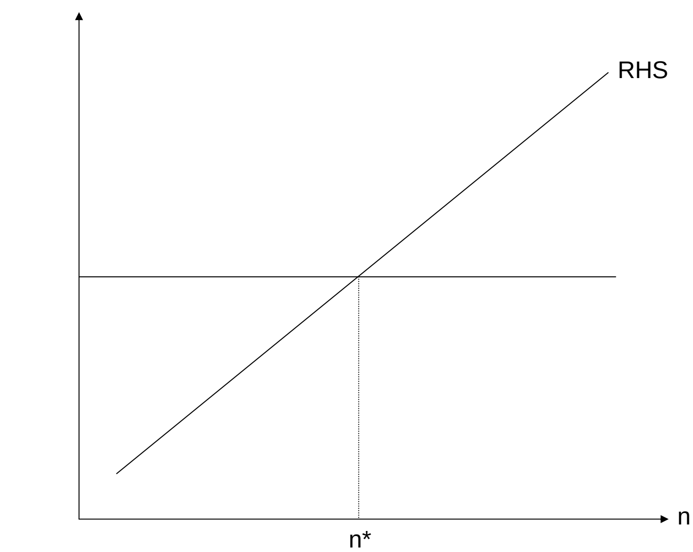

Figure 1: Unique Equilibrium with Free Substitution

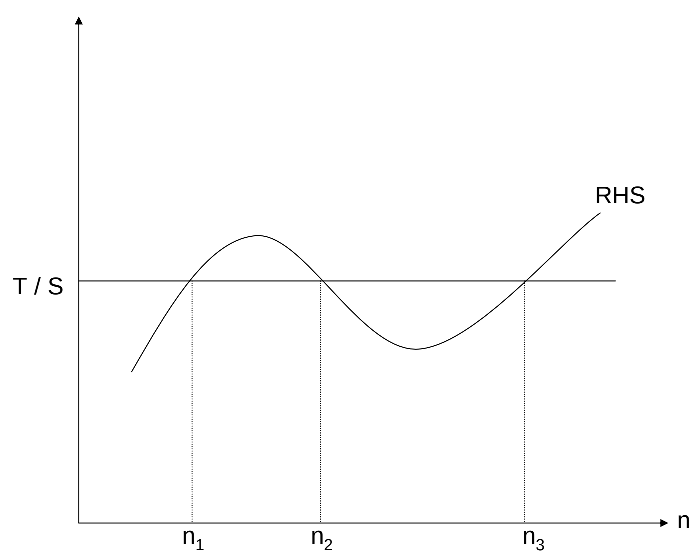

Figure 2: Equilibria with Limited Substitution

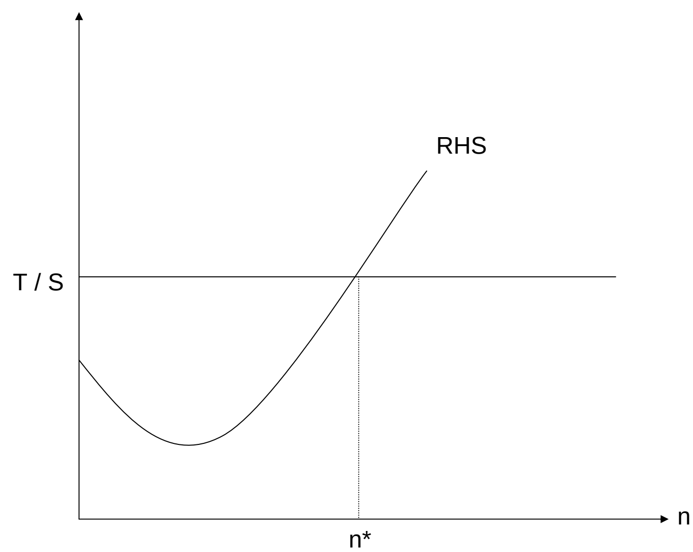

Figure 3: Unique Equilibrium with Free Substitution. A CES Example

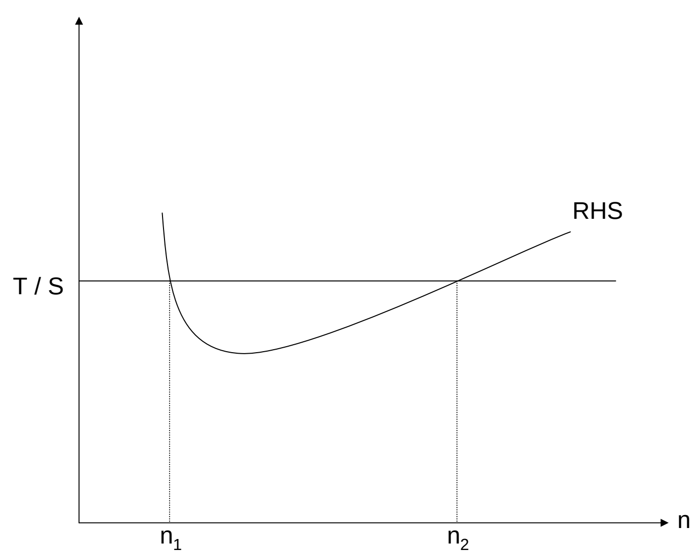

Figure 4: Equilibria with Limited Substitution. A CES Example