Tax Enforcement, Technology, and the Informal Sector

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Abstract

Theoretical models of the informal sector mostly assume—or end up with—a positive correlation between a measure of taxes and the size of the informal sector. However, some recent empirical studies associate higher taxes with a smaller informal sector size. In this paper, we build a theoretical framework—an extension to a two-sector growth model—which allows us to unravel the negative correlation between informal sector size and taxes. We find that (a) a higher degree of tax enforcement, (b) a higher productivity of formal sector households, and (c) a lower physical capital depreciation rate make for a negative relation between these variables. Our results suggest that enforcement and technological factors are likely candidates to account for this relationship.

Keywords: informal sector; tax enforcement; productivity.
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1 Introduction

Theoretical models of the informal sector mostly assume—or end up with—a positive correlation between a measure of taxes (e.g. the level of tax burden) and the size of the informal sector. (Some examples include the papers by Rauch [16], Loayza [14], Fortin et al. [8], Ihrig and Moe [11], Busato and Chiarini [4], and Amaral and Quintin [1].) This result seems intuitive since higher tax rates may create incentives for people to avoid them—and one way of accomplishing this goal is participating in the informal sector. In particular, the positive relation is immediate in a two-sector neoclassical growth model with formal and informal sectors, in a specialization in which taxes are exogenous and informal sector entities have the option to avoid paying taxes (or pay a smaller fraction than in the formal sector).[1] The problem with this theoretical result is that it is not fully supported by the data. Even though earlier studies on informality (such as Frey and Pommerehne [9], Tanzi [21], and Schneider [17, 18]) found evidence supporting the positive correlation between taxes and the extent of informality, recent studies utilizing larger cross-section and panel datasets (see Johnson et al. [12], Johnson et al. [13], Friedman et al. [10], Torgler and Schneider [22], Elgin [5], and Elgin and Solis-Garcia [7]) indicate exactly the opposite: higher tax rates are associated with a smaller size of the informal economy. Elgin [5] and Elgin and Solis-Garcia [7] claim that political turnover and public trust in government policy provide the key for understanding the negative relationship between taxes and informal sector size. Different than (but not contrary to) their findings, we find that three additional factors, namely tax enforcement, productivity, and depreciation also play a significant role in the relationship between taxes and informal sector size.

In this paper, we build a theoretical framework—an extension to a two-sector model—that allows us to unravel the negative correlation between informal sector size and taxes. The main contribution of our paper is to suggest factors that lead to this negative relationship. (For our purposes, the tax burden and a measure of tax rates are equivalent and we will use these terms interchangeably in what follows, unless otherwise specified. See Section 4 for details.) More specifically, we use a two-sector dynamic general equilibrium model where the representative household faces a decision of supplying labor in the formal and informal sectors. Informal sector output depends on labor only whereas formal sector output uses both physical capital and labor. In addition, the formal sector is taxed at a given rate for the household whereas the informal sector can only be taxed imperfectly. Given the decision process of the household, we endogenize the tax rate by allowing the government to determine the rate that maximizes the tax revenue. We then proceed to perform a numerical, steady-state comparative statics exercise in two steps. First, we examine how several (exogenous) factors affect the endogenously determined tax rate, and second, how the tax rate in turn affects the informal sector size. (Note that both the tax rate *and* the informal sector size are determined endogenously in the model.) The exogenous factors we consider are the degree of tax enforcement, the total factor productivity (TFP) gap between the formal and the

[1] One recent exception is the paper by Arbex and Turdaliev [2], who characterize optimal fiscal and monetary policy in the presence of an informal sector. However, in their simulations they still end up with a positive correlation between taxes and informal sector size.

informal sectors,[2] and the depreciation rate of the physical capital stock.[3]

For plausible parameter values, our numerical simulations indicate that a higher revenue maximizing tax rate is obtained by (a) a higher degree of tax enforcement, (b) a higher productivity of the formal sector, and (c) a lower depreciation rate of the capital stock. In turn, all these exogenous changes lead to a lower size of the informal economy. Our results suggest enforcement and technological factors are likely candidates that may account for the negative relationship between taxes and informal sector size.

Our paper is distinct in the growing literature on the informal sector as most (if not all) of the existing theoretical frameworks cannot account for the negative relationship between tax rates and the size of the informal sector. Some of the empirical papers mentioned above deserve additional discussion as they are more closely related to our paper. Both Johnson et al. [12] and Johnson et al. [13] use different sets of countries in their empirical analyses; however, both papers conclude that tax rates are negatively correlated with the size of the informal sector. The first paper provides a very simple model in which the only two stable equilibria of the model are either an entirely formal or an entirely informal economy. However, contrary to their empirical findings, their model implies a positive relationship between the tax rates and the size of the informal sector. The second paper claims that both administration of taxes and regulatory discretion play key roles in this result and, once composite indices of both tax rates and quality of tax administrations are considered, they find that these indices are positively correlated with the size of the informal sector. However, the quality indices they use are largely based on subjective evaluations of certain experts and institutions and therefore prone to measurement errors and endogeneity issues.

Friedman et al. [10] suggest that the positive correlation might have been caused by several institutional factors such as corruption and bureaucratic quality. They find that increasing tax rates by 1% implies that the share of the unofficial economy falls by 9.1%. Controlling for several variables and instrumenting on others reduces the value by half, but the negative tax coefficient remains significant. They conclude that this result probably arises because higher tax rates generate revenue that provides productivity-enhancing public goods, a strong legal environment, and low corruption. However, they only consider the production side of the economy and their partial equilibrium model only focuses on the corruption part of the story. More recently, Aruoba [3] develops a general equilibrium model where the key factor creating the variation in taxes and the size of the shadow economy is the quality of institutions; more specifically, the degree of tax auditing by the government. Accordingly, changes in these factors could let the businesses hide their activities from the government, which by reducing the tax revenues and harming the quality of public administration further reduces a firm's incentives to remain formal.

The rest of our paper is organized as follows. In Section 2 we follow Elgin [5] and employ cross-section, static, and dynamic panel data techniques to show that the negative relationship between various measures of tax rates and the size of the informal sector is significant

[2] To be precise, the *level* of the formal sector's TFP; as we take the informal sector's TFP as fixed, the level of the formal sector's TFP effectively determines the size of the gap.

[3] This particular choice of variables was derived after thinking of real-world objects that we could measure and match to objects in our model, and that have sufficient cross-country variation to obtain different sizes of the informal sector. Details on how these variables are measured in the data and defined in our model can be obtained in sections 2 and 3.

and robust. In addition, econometric analysis explores the relationship between countries' characteristics—enforcement, productivity, and depreciation—to a measure of taxes and to the size of the informal sector. In turn, these relationships are accounted by our model, which together with the definition and characterization of equilibria, is presented in Section 3. Numerical results follow in Section 4. Finally, implications of our work and some concluding remarks are discussed in Section 5.

2 Data and empirical analysis

This section investigates the empirical relationship between taxes and informal sector size. We first describe the data and then present the result of several (panel) regressions that highlight the relationship between taxes and the size of the informal sector.

2.1 Data sources

Informal sector size The literature suggests several ways to estimate the size of the informal economy; Schneider [19] gives a detailed account of these different methods. Our empirical analysis uses the informal economy estimates reported by Schneider et al. [20], which is a data panel of 152 countries in a time span of 9 years between 1999 and 2007.[4]

Taxes As a measure of taxes we use tax burden data (defined as the ratio of the tax revenue to GDP) which we take from the Government Finance Statistics of the IMF.[5] Several other tax indicators (such as tax rates on income, profits and capital gains from the World Development Indicators[6] or the fiscal freedom index from the Heritage Foundation[7]) have also been examined; our results do not depend on whether we use the tax burden or official tax rates.[8]

Main control variables The model we will be presenting in the next section is based on three key exogenous variables that potentially affect the relationship between taxes and informal sector size. These are the depreciation rate of physical capital, the formal sector's (measured) TFP, and the degree of tax enforcement.

To verify whether these variables empirically play a role in this context, we construct proxies for these variables as described below. First, to create the TFP series for the formal sector, we assume that output in the formal sector is produced using a Cobb-Douglas technology specification of the form

$$Y_{Ft} = \theta_{Ft} K_t^{\alpha} N_{Ft}^{1-\alpha}, \tag{1}$$

[4] Our results do not change qualitatively when we use the informal economy estimates reported by Elgin and Oztunali [6].

[5] See `<http://www.imf.org/external/pubs/ft/gfs/manual/gfs.htm>`.

[6] See `<http://data.worldbank.org/data-catalog/world-development-indicators>`. The data used was retrieved on April 10, 2011.

[7] See `<http://www.heritage.org/index>`.

[8] Regression results using statutory taxes rather than the tax burden are available upon request. Also see Elgin [5] for a discussion of the choice of the relevant tax indicator in this context.

where θ_{Ft} is formal sector TFP, K_t is the stock of physical capital, and N_{Ft} is formal sector employment. With measured values for Y_{Ft}, K_t, and N_{Ft},[9] it is easy to back out TFP from equation (1) via

$$\theta_{Ft} = \frac{Y_{Ft}}{K_t^{\alpha} N_{Ft}^{1-\alpha}}.$$

The series for depreciation is obtained from the World Development Indicators, where it is defined as the consumption of fixed capital as given by the replacement value of capital used up in the production process.[10]

Finally, to proxy tax enforcement we use two separate variables. The first one, an index on law and order, is obtained from the International Country Risk Guide (ICRG) provided by Political Risk Services.[11] The second one, the inverse of seignorage,[12] is calculated using raw data from the World Development Indicators and the International Financial Statistics.[13]

Other control variables We also include GDP per capita, a corruption control index, and the real interest rate. We obtain GDP per capita data from the Penn World Tables and interest rate data from the World Development Indicators.[14] Finally, the data on corruption control is obtained from ICRG.[15]

Summary statistics Summary statistics of all variables are provided in Table 1. Our data is an unbalanced panel with 152 countries and a time horizon of 9 years, from 1999 to 2007.

2.2 Taxes and the informal sector

To illustrate the relationship between informal sector size and taxes, we use a simple plot of informal sector size and tax burden in a cross-section—seen in Figure 1 below—which clearly indicates a negative relationship between these variables.[16]

[9] The capital stock series K_t series is constructed using the perpetual inventory method and the investment data series from the Penn World Tables (PWT; see <http://pwt.econ.upenn.edu>). We assume a constant rate of depreciation of 8% as standard in the real business cycle literature. The number of workers is also taken from PWT. In this specification we also assume that $\alpha = 0.33$, so that it matches with the parameter value we select in the next section. See Table 7 for details.

[10] Notice that the mean of the depreciation series for the panel data is about 0.08, which is also the constant value we use to construct the capital stock series. By not allowing the value to vary across countries and across time, we also ensure that the variation of the physical capital does not originate from the variation of the depreciation rate.

[11] See <http://www.prsgroup.com/ICRG.aspx>.

[12] Following Ihrig and Moe [11]—along with many others—we use the inverse of seignorage as a proxy for tax enforcement. The idea here is that countries that rely more on seignorage revenue are the ones with reduced tax enforcement. We define seignorage as the ratio of the increase in the base money to total government revenue.

[13] See <http://www.imfstatistics.org/imf/about.asp>.

[14] These are the most-widely used control variables in the empirical literature on informality.

[15] Our results do not change qualitatively when we use corruption control measures from Transparency International.

[16] It is worth emphasizing the distinction between the tax burden and various statutory tax rates: the tax burden is defined as the ratio of total tax revenues to GDP. One may suspect that the negative relation

Table 1: Summary statistics.

	Mean	Std. Dev.	Minimum	Maximum
Tax burden[a]	29.12	14.52	0.06	70.22
Informal sector size[a]	34.40	13.46	8.60	68.35
Law and order	3.89	1.35	0.50	6.00
(Inverse) Seignorage	0.26	0.35	0.04	2.50
Depreciation	0.08	0.06	0.04	0.21
Formal productivity	1.89	1.50	1.75	2.50
Real interest rate	0.07	0.10	-0.14	0.86
GDP per capita[b]	12.09	12.99	0.31	68.29
Corruption control	2.78	1.22	0.00	6.00

These are cross-section summary statistics of the panel averages.
[a] In percent. [b] In thousand US$, Geary-Khamis PPP-adjusted.

In addition, we run a set of simple panel regressions, the results of which are reported in Table 2. The equation we estimate in a static panel data setting is given by

$$IS_{i,t} = \beta_0 + \beta_1 IS_{i,t-1} + \beta_2 \text{Tax}_{i,t} + \sum_{k=3}^{n} \beta_k X_{k_{i,t}} + \theta_i + \gamma_t + \epsilon_{i,t}, \tag{2}$$

where the matrix $X_{k_{i,t}}$ contains control variables and γ_t and θ_i are year and country fixed effects.

In the first six columns of Table 2, we regress informal sector size on the tax burden and additional control variables such as corruption control, real interest rate, GDP per-capita, and one-period lagged informal sector size. In the sixth regression we also report the results of a dynamic panel data regression using the Arellano-Bond estimator, where we use the one-period lagged informal sector size among the independent variables. The consistently negative coefficient of tax burden in these regressions indicates a robust negative correlation between tax burden and the informal sector size.[17] However, this result changes when we include measures of tax enforcement (i.e. law and order index or inverse seigniorage), total factor productivity of formal output and depreciation rate of physical capital. That is, when either of these four variables is controlled for, the robust negative relationship between tax burden and informal sector size either ceases to be significant or turns out to be significantly

between the tax burden and the informal sector may arise simply because a larger informal economy implies a smaller tax base; however, a larger informal economy also implies a lower official GDP, as only imperfect estimates of the informal economy are included in the national income calculations. Moreover, as Elgin [5] clearly demonstrates, the negative correlation is also evident between statutory tax rates and the size of the informal sector.

[17] We should notice that we do not make any arguments regarding the direction of causality between informality and taxes but only investigate how these two variables are associated with each other. Nevertheless, we also have conducted a simultaneous equation regression where we regress taxes on informal sector size as well as informal sector size on taxes in a two-equation system. The negative correlation between the two variables still persists in the system estimations, yet we find no evidence supporting the presence of two-way causality. For space constraints we only mention this in a footnote and refer the interested reader to the corresponding author.

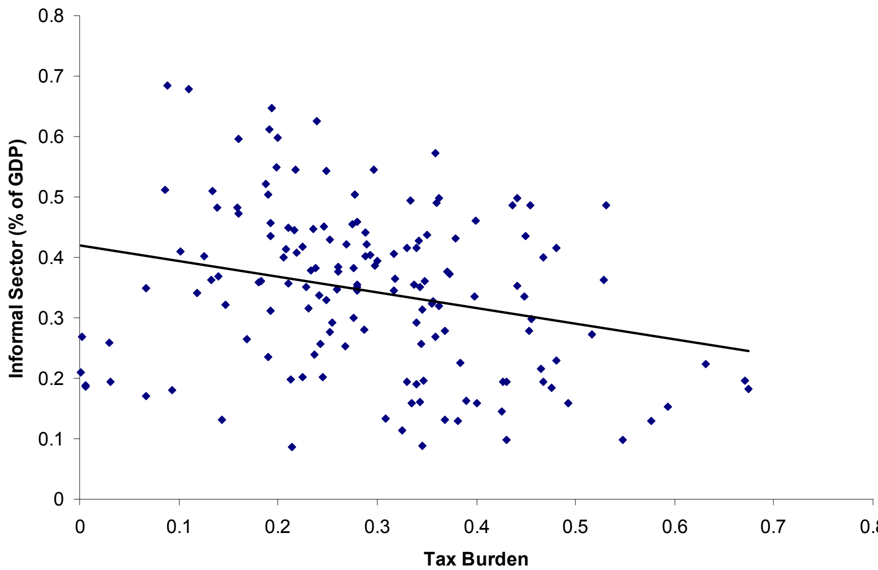

Figure 1: Informal Sector and Tax Burden

positive. This result indicates that tax enforcement, formal productivity, and depreciation rate are the driving forces behind the relationship between taxes and informal sector size.[18]

2.3 Further empirical evidence

We now investigate how changes in enforcement, productivity, and depreciation affect the relationship between taxes and size of the informal sector. We estimate the following system using both 3SLS and GMM:

$$\begin{aligned} \text{Tax}_{i,t} &= \alpha_0 + \alpha_1 \text{Tax}_{i,t-1} + \alpha_2 Z_{i,t} + \theta_i + \gamma_t + \epsilon_{i,t} && (3) \\ IS_{i,t} &= \beta_0 + \beta_1 IS_{i,t-1} + \beta_2 \text{Tax}_{i,t} + \sum_{k=3}^{n} \beta_k X_{k_{i,t}} + \theta_i + \gamma_t + \nu_{i,t}, && (4) \end{aligned}$$

where $Z_{i,t}$ corresponds to either tax enforcement (measured with the law and order index or inverse seignorage), productivity, or depreciation. The term $X_{k_{i,t}}$ in the second equation includes control variables that may be correlated with informal sector size.

2.3.1 Enforcement

First, we check the effect of tax enforcement—proxied by the law and order index—on the tax burden and informal sector size. Table 3 presents the results of the system estimation.

[18] We have also run regressions (with law and order, inverse seignorage, productivity, and depreciation) using the GMM estimator and the lagged informal sector size among the independent variables and obtained qualitatively similar results. Additionally, we also checked for the direction of causality by running a simultaneous equation regression using these four variables and informal sector size and found no evidence supporting the presence of two-way causality. Due to space constraints we do not report these and refer the interested reader to contact the corresponding author.

Table 2: Informal sector size (IS) and tax burden.

	FE	FE	FE	FE	FE	GMM	FE	FE	FE	FE
Tax burden	-0.09***	-0.12***	-0.14***	-0.05**	-0.12**	-0.04**	0.03*	0.04	0.04	0.04**
	(0.03)	(0.04)	(0.04)	(0.02)	(0.06)	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)
Corruption		-2.76***	-3.16***	-3.18***	-3.19***	-2.99***	-2.42**	-2.38**	-3.19***	-3.05***
		(0.53)	(0.73)	(0.96)	(1.04)	(1.01)	(1.11)	(1.17)	(1.06)	(1.04)
Interest rate			0.20***	0.21***	0.20***	0.17**	0.19**	0.18**	0.16*	0.15*
			(0.04)	(0.05)	(0.04)	(0.08)	(0.09)	(0.09)	(0.09)	(0.08)
GDP per capita				-0.1***	-0.1***	-0.02**	-0.02**	-0.02**	-0.02**	-0.02**
				(0.03)	(0.04)	(0.01)	(0.01)	(0.01)	(0.1)	(0.01)
IS (lag)					1.01**	0.89*				
					(0.45)	(0.50)				
Law and order							-1.10**			
							(0.44)			
Inverse seignorage								-7.10**		
								(2.99)		
Productivity									-1.27**	
									(0.60)	
Depreciation										10.99***
										(2.45)
R-squared	0.29	0.34	0.38	0.49	0.60		0.57	0.53	0.52	0.62
Obs.	1,167	965	885	883	726	622	883	883	883	883
F-Test	0.00	0.00	0.01	0.00	0.00	0.02	0.00	0.00	0.00	0.00
Hansen J-Test						0.36				
AR(2) Test						0.24				

All panel regressions include year and country fixed effects. Standard errors are reported in parentheses. 1, 5, and 10% confidence levels are denoted by ***, **, and *, respectively.

Panel [A] shows that higher tax enforcement (i.e. higher values for the index) is associated with a higher tax burden; panel [B] shows next that a higher tax burden is associated with a smaller informal sector size.

Next, we check the effect of tax enforcement—now proxied by inverse seignorage—on the tax burden and informal sector size. Table 4 presents the results of the system estimation; as was the case in Table 3, panel [A] shows that higher tax enforcement (i.e. higher values for inverse seignorage) is associated with a higher tax burden and panel [B] shows next that a higher tax burden is associated with a smaller informal sector size.

2.3.2 Productivity

Table 5 presents the results of the system estimation made by using TFP as the relevant exogenous variable. From panel [A], we observe that higher productivity is associated with higher taxes; panel [B] shows that higher taxes are in turn associated with a smaller informal sector size.

Table 3: Regressions with law and order index as enforcement.

	3SLS	3SLS	3SLS	GMM	GMM	GMM
[A] Tax burden						
Law and order	0.64**	0.62**	0.61**	0.61**	0.65**	0.65**
	(0.30)	(0.31)	(0.30)	(0.30)	(0.33)	(0.33)
Tax burden (lag)		0.80***	0.89***		0.85***	0.87***
		(0.27)	(0.22)		(0.21)	(0.22)
R-squared	0.41	0.84	0.85	0.44	0.83	0.83
Observations	898	778	778	898	778	778
χ-squared	77.12	69.19	91.02	79.19	69.19	54.01
[B] Informal sector size						
Tax burden	-0.11***	-0.12**	-0.19**	-0.10**	-0.14**	-0.16**
	(0.03)	(0.06)	(0.08)	(0.05)	(0.07)	(0.07)
GDP per capita		-0.04**	-0.04**		-0.04**	-0.04**
		(0.02)	(0.02)		(0.02)	(0.02)
Corruption		-3.60***	-3.44**		-3.51**	-3.21**
		(1.14)	(1.70)		(1.29)	(1.60)
Interest rate		0.22***	0.21***		0.21**	0.21**
		(0.05)	(0.06)		(0.10)	(0.10)
Informal sector size (lag)			0.90**			0.91**
			(0.46)			(0.45)
R-squared	0.11	0.45	0.79	0.12	0.48	0.81
Observations	1,022	1,020	889	1,022	1,020	889
χ-squared	109.11	103.21	97.15	108.11	98.10	141.23

All panel regressions include year and country fixed effects. Standard errors are reported in parentheses. 1 and 5% confidence levels are denoted by *** and **, respectively.

2.3.3 Depreciation rate

Table 6 presents the results of the system estimation when using the depreciation rate of physical capital as the relevant exogenous variable. From panels [A] and [B], we observe that a higher rate of depreciation is associated with lower taxes, which in turn is associated with a larger informal sector size.

2.4 Summing up

As was mentioned in the introduction, theoretical models of the informal sector assume that higher taxes imply a larger informal sector size. The results contained in this section demonstrate that this assumption is not supported by the data: the first six columns of the first row in Table 2 shows that the relationship—equation (2)—is negative and robust to any of the specifications presented therein. However, the relationship changes to being positive, only when we include measures of enforcement (inverse seignorage and law and order), productivity and depreciation. Moreover, tables 3-6 indicate that the negative relationship is extremely robust; changes via (a) including lagged values of taxes or informal sector among

Table 4: Regressions with inverse seignorage as enforcement.

	3SLS	3SLS	3SLS	GMM	GMM	GMM
[A] Tax burden						
Inverse seignorage	1.94**	1.92**	1.90**	1.90**	1.95**	1.95**
	(0.91)	(0.92)	(0.91)	(0.89)	(0.92)	(0.93)
Tax burden (lag)		0.81***	0.87***		0.84***	0.84***
		(0.24)	(0.21)		(0.22)	(0.23)
R-squared	0.40	0.80	0.80	0.41	0.81	0.81
Observations	811	691	691	811	691	691
χ-squared	104.20	99.38	89.24	111.33	134.15	105.12
[B] Informal sector size						
Tax burden	-0.12***	-0.12**	-0.18**	-0.11**	-0.13**	-0.16**
	(0.04)	(0.05)	(0.09)	(0.05)	(0.06)	(0.08)
GDP per capita		-0.04**	-0.04**		-0.04**	-0.04**
		(0.02)	(0.02)		(0.02)	(0.02)
Corruption		-3.51***	-3.43**		-3.50**	-3.51**
		(1.14)	(1.70)		(1.29)	(1.60)
Interest rate		0.21***	0.20***		0.22**	0.21**
		(0.05)	(0.06)		(0.10)	(0.10)
Informal sector size (lag)			0.90**			0.91**
			(0.46)			(0.45)
R-squared	0.11	0.45	0.79	0.12	0.48	0.81
Observations	1,022	1,020	889	1,022	1,020	889
χ-squared	107.10	104.19	95.32	106.88	101.03	109.99

All panel regressions include year and country fixed effects. Standard errors are reported in parentheses. 1 and 5% confidence levels are denoted by *** and **, respectively.

the independent variables in respective equations, (b) estimating the system with 3SLS or GMM, or (c) including other determinants of the informal sector in our regressions do not modify our empirical findings.

The first-step empirical relationships—equation (3), relating the tax burden and our exogenous variables—can summarized as being positive for tax burden and enforcement, productivity, and negative for tax burden and depreciation. The second-step empirical relationships—equation (4)—show that tax burden and informal sector size have a negative relationship when using enforcement, productivity or depreciation as instruments.

From the above, it follows that these variables are likely candidates to account for the relationship. In summary, these results indicate that a higher degree of enforcement, a higher productivity, and a smaller depreciation of physical capital stock are associated with a larger tax burden as well as a smaller informal sector size. That is, better enforcement, higher productivity and smaller depreciation potentially create a push effect from the informal sector, as they reduce the return from informality, thereby reducing the room for avoiding taxes in the economy. With less room to avoid taxes, the government is now able to levy a relatively larger tax rate. As labor is pulled towards the formal sector, a higher tax rate pushes it to the informal one. At the end the pull is stronger than the push and informal

Table 5: Regressions with productivity.

	3SLS	3SLS	3SLS	GMM	GMM	GMM
[A] Tax burden						
Productivity	1.22**	1.21**	1.21**	1.23**	1.22**	1.21**
	(0.60)	(0.61)	(0.59)	(0.59)	(0.59)	(0.58)
Tax burden (lag)		0.81***	0.88***		0.84***	0.89***
		(0.24)	(0.23)		(0.23)	(0.25)
R-squared	0.38	0.85	0.85	0.38	0.85	0.85
Observations	898	778	778	898	778	778
χ-squared	99.99	105.10	103.28	104.07	97.66	101.30
[B] Informal sector size						
Tax burden	-0.11**	-0.15**	-0.15**	-0.10**	-0.12**	-0.12**
	(0.05)	(0.07)	(0.07)	(0.05)	(0.06)	(0.06)
GDP per capita		-0.04**	-0.04**		-0.04**	-0.03**
		(0.02)	(0.02)		(0.02)	(0.02)
Corruption		-3.57***	-3.49**		-3.56***	-3.20**
		(1.14)	(1.70)		(1.29)	(1.60)
Interest rate		0.20***	0.20***		0.20**	0.20**
		(0.05)	(0.06)		(0.10)	(0.10)
Informal sector size (lag)			0.91**			0.91**
			(0.46)			(0.45)
R-squared	0.12	0.49	0.80	0.12	0.48	0.80
Observations	1,022	1,020	889	1,022	1,020	889
χ-squared	103.29	100.01	99.49	111.74	108.65	120.12

All panel regressions include year and country fixed effects. Standard errors are reported in parentheses. 1 and 5% confidence levels are denoted by *** and **, respectively.

sector size is reduced.

To understand the economic mechanism behind this observation, we need to rely on a theoretical model. To this end, we will use a two-sector dynamic general equilibrium model in the next section.

3 A model of the informal sector

We now present a theoretical model of the informal sector. As stated above, we follow the framework of Ihrig and Moe [11] but add the following variation: we endogenize the tax rate by allowing the government to determine the rate that maximizes tax revenue. We now describe the model in detail.

Agents There are two infinitely-lived agents in the economy: a representative household and a government.

Table 6: Regressions with depreciation rate.

	3SLS	3SLS	3SLS	GMM	GMM	GMM
[A] Tax burden						
Depreciation	-5.79**	-5.71**	-5.59**	-5.71**	-5.75**	-5.56**
	(2.40)	(2.42)	(2.87)	(2.49)	(2.48)	(2.88)
Tax burden (lag)		0.81***	0.83***		0.82***	0.84***
		(0.25)	(0.23)		(0.22)	(0.22)
R-squared	0.08	0.64	0.64	0.09	0.67	0.68
Observations	898	778	778	898	778	778
χ-squared	99.20	101.30	107.52	105.70	134.54	110.11
[B] Informal sector size						
Tax burden	-0.18**	-0.19**	-0.21***	-0.18**	-0.19**	-0.22**
	(0.09)	(0.09)	(0.07)	(0.09)	(0.09)	(0.06)
GDP per capita		-0.03**	-0.03**		-0.04**	-0.04**
		(0.01)	(0.01)		(0.02)	(0.02)
Corruption		-3.40***	-3.38**		-3.41***	-3.41**
		(1.14)	(1.70)		(1.29)	(1.60)
Interest rate		0.26***	0.25***		0.23**	0.23**
		(0.05)	(0.06)		(0.10)	(0.10)
Informal sector size (lag)			0.92**			0.92**
			(0.46)			(0.45)
R-squared	0.13	0.46	0.89	0.13	0.49	0.89
Observations	1,022	1,020	889	1,022	1,020	889
χ-squared	107.61	102.81	101.20	98.71	92.14	104.88

All panel regressions include year and country fixed effects. Standard errors are reported in parentheses. 1 and 5% confidence levels are denoted by *** and **, respectively.

Endowments The representative household has a strictly positive endowment of K_0 initial units of physical capital and T units of time every period.

Technology The household has access to two productive technologies, denoted formal and informal. The formal technology uses capital and labor inputs to produce final output, following the specification

$$Y_{Ft} = \theta_{Ft} K_t^{\alpha} N_{Ft}^{1-\alpha}.$$

In the above, Y_{Ft} denotes the units of final output produced using the formal technology, θ_{Ft} is a stationary technology disturbance exclusive to the formal technology, and N_{Ft} stands for the units of time that the household devotes to operating the technology. In contrast, the informal technology uses the labor input to produce final output following

$$Y_{It} = \theta_{It} N_{It}^{\gamma},$$

where Y_{It} denotes the units of final output produced using the informal technology, θ_{It} is a stationary technology disturbance exclusive to the informal technology, and N_{It} stands for the units of time that the household supplies to the informal sector.

We assume that the units of final output produced with the formal technology are perfectly observable. At a cost of zero, the household can attempt to hide the units produced via the informal technology. Final output can be transformed one-to-one into units of a consumption good (C_t) or an investment good (X_t), such that

$$C_t + X_t = Y_{Ft} + Y_{It}.$$

Finally, capital accumulates following the law of motion

$$K_{t+1} = (1 - \delta)K_t + X_t,$$

where $\delta \in [0, 1]$ is the depreciation rate of the physical capital stock.

Preferences The representative household ranks among infinite sequences of consumption via the ordering

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$$

where $\beta \in (0, 1)$ is a discount factor and where we assume that the instantaneous utility function U is strictly increasing and strictly concave.[19] While we don't impose a particular preference ordering for the government, we do make the assumption that it chooses a value for the tax rate such that total (expected) tax revenue is maximized. We further impose the requirement that all the tax revenue is spent in a non-productive way, such that neither the household's utility nor the formal or informal technology is made more productive.[20]

Government The government has the ability to impose a tax rate $\tau_t \in [0, 1]$ over the production of final output, as well as a tax surcharge $\tau_S > 0$ over informal output.[21] While the government can levy taxes over formal output without problem, it can only enforce payment of taxes on informal output (this is, the tax rate τ_t plus the surcharge τ_S) with a probability $\rho \in [0, 1]$, which is exogenous and known by the representative household.[22] It follows that the expected tax liability associated with the informal technology is $\rho(\tau_t + \tau_S)\theta_{It} N_{It}^{\gamma}$.

3.1 Competitive equilibrium

An equilibrium is easy to define:

[19] We assume an instantaneous utility specification with inelastic labor supply. This assumption is not relevant to our results.

[20] It's easy to rationalize this behavior by thinking of a government—a group of bureaucrats, for example—which are set to maximize their rents by stealing as much as possible: the higher the tax revenue, the higher the amount of rents they can extract.

[21] The tax surcharge is a technical requirement of our model; as long as τ_S takes a positive value our results are qualitatively similar. We assume that τ_S is constant over time.

[22] One interpretation of the variable ρ is the probability of an audit to the household.

Definition 3.1. Given a level of enforcement ρ and a tax surcharge τ_S, a (tax-distorted) competitive equilibrium consists of sequences of household allocations $\{C_t, X_t, N_{Ft}, N_{It}\}_{t=0}^{\infty}$ and government policy $\{\tau_t, G_t\}_{t=0}^{\infty}$ such that

1. The sequences $\{C_t, X_t, N_{Ft}, N_{It}\}_{t=0}^{\infty}$ solve the representative household's problem

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(C_t) && (5) \\ \text{s.t.} \quad & C_t + X_t = (1-\tau_t)\theta_{Ft} K_t^{\alpha} N_{Ft}^{1-\alpha} + (1-\rho(\tau_t+\tau_S))\theta_{It} N_{It}^{\gamma}, \text{ all } t \\ & K_{t+1} = (1-\delta)K_t + X_t, \text{ all } t \\ & N_{It} + N_{Ft} = T, \text{ all } t. \end{aligned}$$

2. The sequences $\{\tau_t, G_t\}_{t=0}^{\infty}$ make the government's budget constraint

$$G_t = \tau_t \theta_{Ft} K_t^{\alpha} N_{Ft}^{1-\alpha} + \rho(\tau_t + \tau_S)\theta_{It} N_{It}^{\gamma} \tag{6}$$

hold every period.

In what follows, we specialize the instantaneous utility function U to a logarithmic functional form, such that $U(C_t) = \log C_t$.

3.2 Steady-state characterization

(Throughout the paper, model variables without time subscripts denote steady state values.) Standard methods allow us to derive the household's Euler equations; in the steady state, these take form

$$\gamma(1-\rho(\tau+\tau_S))\theta_I N_I^{\gamma-1} = (1-\alpha)(1-\tau)\theta_F K^{\alpha} N_F^{-\alpha} \tag{7}$$

and

$$K = \left[\frac{1-\beta(1-\delta)}{\alpha\beta(1-\tau)\theta_F}\right]^{\frac{1}{\alpha-1}} N_F. \tag{8}$$

Combine equations (7) and (8); after some algebra, informal sector labor equals

$$N_I = \left[\frac{(1-\alpha)(1-\tau)\theta_F}{\gamma(1-\rho(\tau+\tau_S))\theta_I}\right]^{\frac{1}{\gamma-1}} \left[\frac{1-\beta(1-\delta)}{\alpha\beta(1-\tau)\theta_F}\right]^{\frac{\alpha}{(1-\alpha)(1-\gamma)}} \tag{9}$$

and using the household's time constraint, it follows that

$$N_F = T - N_I. \tag{10}$$

As the current model takes taxes as exogenously given, there is still a positive correlation—similar to that in Ihrig and Moe [11]—between taxes and the size of the informal sector.

3.3 Revenue maximizing (steady-state) tax rates

We now add a crucial modification to the model: instead of taking the key government's policy variable τ_t to be exogenously determined, we allow the government to choose its value in a way that it maximizes tax revenue, considering that the households will behave in a competitive way once they observe the government's decision. We use Nash equilibrium as our equilibrium notion.

Definition 3.2. Given a level or enforcement ρ and a tax surcharge τ_S, a (Nash) equilibrium with strategic government behavior consists of sequences of household allocations $\{\hat{C}_t, \hat{X}_t, \hat{N}_{Ft}, \hat{N}_{It}\}_{t=0}^{\infty}$ and government policy $\{\hat{\tau}_t, \hat{G}_t\}_{t=0}^{\infty}$ such that

1. The sequences $\{\hat{C}_t, \hat{X}_t, \hat{N}_{Ft}, \hat{N}_{It}\}_{t=0}^{\infty}$ solve the household's problem (5).

2. The sequence $\{\hat{\tau}_t\}_{t=0}^{\infty}$ solves the problem

$$\max_{\{\tilde{\tau}_t\}_{t=0}^{\infty}} \quad \tilde{\tau}_t \theta_{Ft} \hat{K}_t^{\alpha} \hat{N}_{Ft}^{1-\alpha} + \rho(\tilde{\tau}_t + \tau_S)\theta_{It} \hat{N}_{It}^{\gamma}$$

   and the sequence $\{\hat{G}_t\}_{t=0}^{\infty}$ satisfies

$$\hat{G}_t = \hat{\tau}_t \theta_{Ft} \hat{K}_t^{\alpha} \hat{N}_{Ft}^{1-\alpha} + \rho(\hat{\tau}_t + \tau_S)\theta_{It} \hat{N}_{It}^{\gamma} \tag{11}$$

   for every t.

Equations (8)-(10) are still valid under Definition 3.2.[23] In the rest of the paper we will work with the steady state of the model as derived above.

3.3.1 Finding the revenue-maximizing tax rate

The right hand side of equation (11) is the closed-form expression for tax revenue, which we will denote by $\mathcal{R}(\hat{\tau})$. The value of $\hat{\tau}$ that maximizes tax revenue is the one that satisfies $\partial\mathcal{R}(\hat{\tau})/\partial\hat{\tau} = 0$. By the chain rule,

$$\frac{\partial \mathcal{R}}{\partial \hat{\tau}} = \theta_F \left\{ \hat{\tau} \left[(1-\alpha) \left(\frac{K}{N_F} \right)^{\alpha} \frac{\partial N_F}{\partial \hat{\tau}} + \alpha \left(\frac{K}{N_F} \right)^{\alpha-1} \frac{\partial K}{\partial \hat{\tau}} \right] + K^{\alpha} N_F^{1-\alpha} \right\} + \rho \theta_I N_I^{\gamma} \left[\frac{\gamma(\hat{\tau} + \tau_S)}{N_I} \frac{\partial N_I}{\partial \hat{\tau}} + 1 \right]. \tag{12}$$

[23] Given this fact, and to avoid unnecessary clutter in the rest of the paper, we will drop the hats in the household allocations, but keep them in the government policy sequences, in order to distinguish them from the equilibrium objects presented in Definition 3.1.

We already have closed-form expressions for $N_I(\hat{\tau})$, $N_F(\hat{\tau})$, and $K(\hat{\tau})$. In addition,[24]

$$\begin{aligned}
\frac{\partial N_I}{\partial \hat{\tau}} &= \frac{N_I}{\gamma - 1}\left[\frac{\rho}{1-\rho(\hat{\tau}+\tau_S)} + \frac{1}{(\alpha-1)(1-\hat{\tau})}\right] \\
\frac{\partial N_F}{\partial \hat{\tau}} &= -\frac{\partial N_I}{\partial \hat{\tau}} \\
\frac{\partial K}{\partial \hat{\tau}} &= K\left[\frac{1}{N_F}\frac{\partial N_F}{\partial \hat{\tau}} + \frac{1}{(\alpha-1)(1-\hat{\tau})}\right].
\end{aligned}$$

Summing up, the revenue maximizing tax rate can be calculated using these expressions along with (12).

4 Numerical results

Here we take the model outlined in Section 3 and use it to generate artificial data to be compared with the results from Section 2. We first present the parameter values we will use, and then outline the results of the numerical exercises and how the artificial data compares to that in the real world.

While we use the tax burden in Section 3, the equivalent object in the model outlined above is a tax rate over formal (and informal, given a positive value for ρ) sector output. Given the particular structure of the tax scheme, there is a one-to-one mapping between the tax rate and the tax burden, so we can use the tax rate and the tax burden interchangeably.

4.1 Parameter values

We take the parameter values contained in Ihrig and Moe [11] as a benchmark. In their paper, they calibrate a set of parameters for Sri Lanka; these parameters are presented in Table 7 below. Recall that in our simulations, the value for the tax rate is endogenously determined (in equilibrium) by the government.[25] We set the value for the tax surcharge τ_S to 0.05; as we have mentioned earlier, the magnitude of the surcharge does not qualitatively change our results.

The values for θ_F and θ_I imply that TFP for the informal technology is about 25 times higher than the formal technology. This fact does not have a direct translation in output, as the formal technology still includes capital input in the production of the final good, whereas the informal technology does not. Therefore, one might attribute this difference to the fact that some of the capital (absent in the informal sector production function) is captured by the TFP parameter of the informal sector.[26]

[24] See the paper's technical appendix for details on these calculations.
[25] As a reference, the value of τ used in their paper is set to 0.093.
[26] See Ihrig and Moe (2004) for a discussion on this.

Table 7: Benchmark parameter values.

Parameter	Value	Description
α	0.33	Capital share in the formal technology
T	100	Available hours
ρ	0	Tax enforcement in the informal sector
θ_F	2.1	Formal sector total factor productivity
θ_I	55.3	Informal sector total factor productivity
γ	0.495	Labor share in the informal technology
δ	0.08	Depreciation rate of capital
β	0.96	Discount factor
τ_S	0.05	Tax surcharge

4.2 Results with an exogenous tax rate

Before presenting the results with an endogenous determination of the tax rate, it is convenient to examine the behavior of the model with exogenous taxes, to better observe the effect of having an endogenous tax rate. More specifically, we use the values of the parameters listed in Table 7 and, using equation (9), we plot the behavior of N_I with respect to an exogenous change in τ. The relationship is shown in Figure 2. As expected, we observe a positive correlation between informal labor and the tax rate.[27]

We could also look at the behavior of informal output rather than time devoted to the informal technology. Yet, since informal output is a (positive) monotone transformation of informal labor time, the qualitative change works in the same direction for both variables. Hence, we do not report results on the behavior of informal output.

4.3 Results with an endogenous tax rate

Here we present the main contributions of our paper. In all exercises, the procedure is similar: we allow an exogenous variable (enforcement, productivity, or depreciation) to change within a certain interval and find the revenue maximizing tax rate to be levied by the government, together with the equilibrium allocations associated to that particular rate. We find two relationships: that of the exogenous variable and the tax rate, and that of the informal labor time and the tax rate (induced in turn by the value of the exogenous variable).

4.3.1 Computational details

To obtain the model data shown in the following subsections we implement the following algorithm. We initialize by creating a discrete grid over the relevant interval for the exogenous variable. In a first step we fix a point in the grid and, for an initial guess of the revenue

[27] We initialize by creating a 100,000 point grid over the tax rate interval $[0, 0.15]$. We fix a point in the grid and calculate the household steady-state allocations $\{N_I, N_F, K\}$ associated with the value of the tax rate. We don't consider equilibrium values where the informal labor time is above 100.

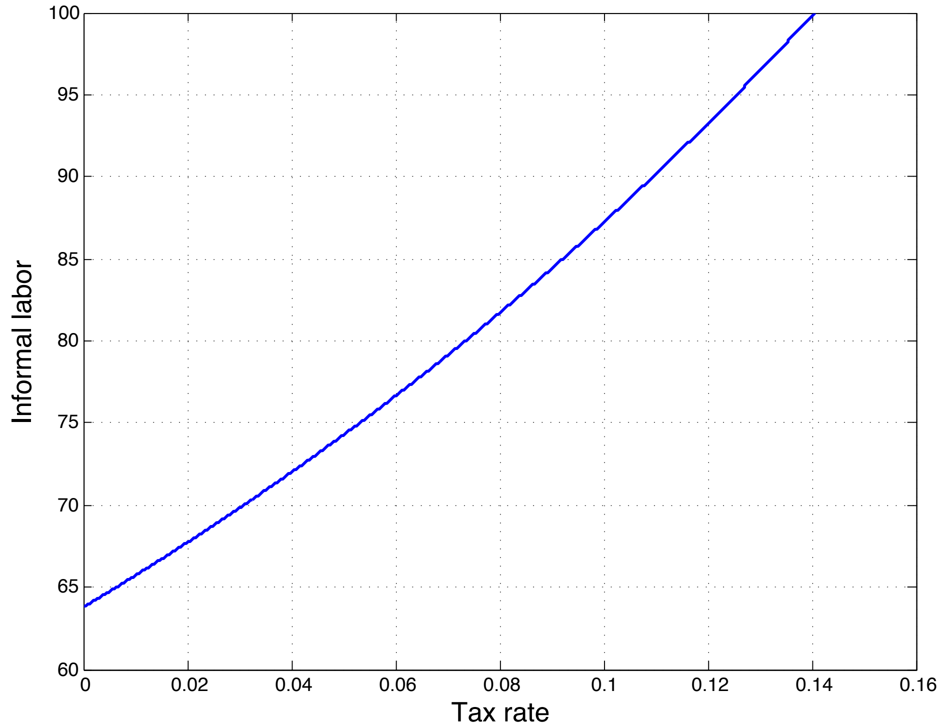

Figure 2: Informal labor and tax rate (exogenously determined case).

maximizing tax rate—call it $\hat{\tau}_0$, we calculate the associated household steady-state allocation $\{N_I, N_F, K\}$. In a second step, given the household allocation, we use a Newton procedure to find $\hat{\tau}_1$, the value of the tax rate that maximizes the government's tax revenue, based on equations (8)-(10) and (12).

To reach the fixed point required for the equilibrium of Definition 3.2 we repeat the first step using the value of $\hat{\tau}_1$ as an initial guess. We stop the iteration once $\|\hat{\tau}_1 - \hat{\tau}_0\|$ is less than some specified tolerance level, at which point we record the equilibrium values of the household allocation, the tax rate, and the associated government tax revenue. The algorithm then jumps to the next point on the exogenous variable grid and the process is repeated until all the points in the grid are covered.

In all of the exercises below, the grid size is 100,000 points and the initial guess for the revenue maximizing tax rate is 0 percent. The tolerance level for the Newton algorithm is 1×10^{-3}. We don't consider equilibrium values where the tax rate is above 100%.

4.3.2 Varying enforcement

We first check how the revenue maximizing tax rate $\hat{\tau}$ varies as we change the enforcement parameter ρ. We allow ρ to vary within the interval $[0, 1]$ and calculate $\hat{\tau}$ following the procedure outlined above.[28]

As shown in Figure 3, the revenue maximizing tax rate increases with enforcement. The intuition behind this result is natural: a government with access to a better tax enforcement

[28] The value of ρ in Table 7 is zero, so our interval covers this particular case.

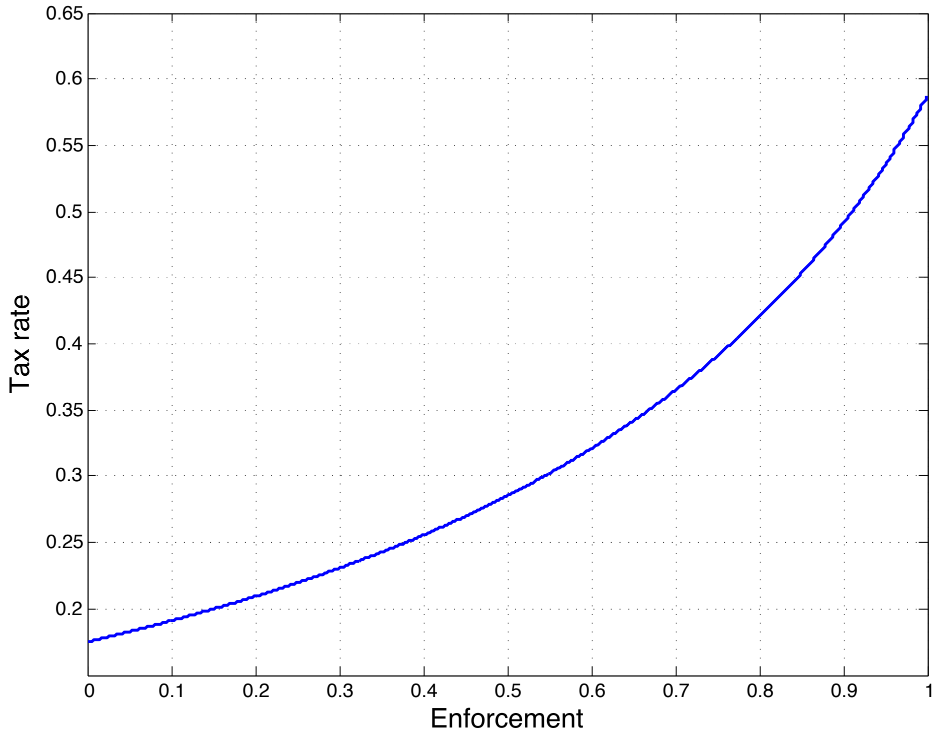

Figure 3: Revenue maximizing tax rate with variable enforcement.

or audit technology on the informal sector is able to impose a higher tax rate and also collect a larger revenue not only from the informal sector but also from the formal sector, as higher tax enforcement will create higher incentives for household to supply more labor in the formal sector. This happens because a larger tax enforcement reduces the payoff of supplying labor in the informal sector, thereby reducing the room for avoiding taxes in the formal sector. With less room to avoid taxes, the government is now able to charge the formal sector at a relatively larger tax rate.

Figure 4 shows the other side of the simulation, namely, the relationship between the amount of time devoted to the informal technology (informal labor) and the tax rate. As tax enforcement (and also the tax) increases, hours devoted to informal sector decrease. Our conclusions coincide with the results from Tables 2 and 3, namely, that there exists a negative correlation between taxes and the size of the informal sector.[29] As a higher enforcement rate pulls hours towards the formal sector, a higher tax rate pushes them to the informal one. At the end the pull is stronger than the push and informal labor is reduced.

4.3.3 Varying productivity

We now see how taxes and labor devoted to the informal sector change when we allow the total factor productivity of the formal technology to vary. Figure 5 illustrates the behavior of the revenue maximizing tax rate with respect to a change in θ_F. In particular, we allow θ_F

[29] We draw the figures with informal labor; however as informal labor and informal sector output move in the same direction, our results also hold for informal sector size as well.

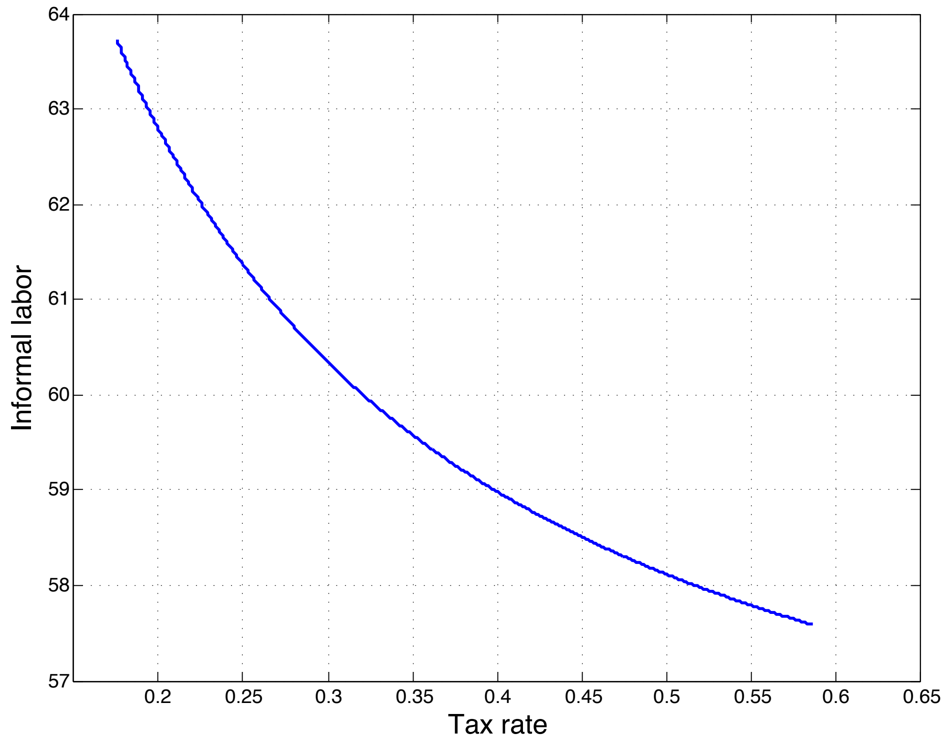

Figure 4: Informal labor and revenue maximizing tax with varying enforcement

to vary between 1.85 and 2.35 (a ± 0.25 band over the base value of $\theta_F = 2.1$).[30] Intuitively, as the formal sector becomes relatively more productive compared to the informal sector, the government can impose a higher tax. This result follows as the representative household, ceteris paribus, has a higher incentive to supply more labor in the formal sector now and the government exploits this incentive by setting a higher tax rate. (Since we use the calibrated value of $\rho = 0$, all the tax is imposed on the formal sector here. However, results do not change qualitatively if we use a different value for ρ.[31])

Even though the tax rate increases, the time devoted to informal labor decreases. As Figure 6 illustrates, we obtain a negative correlation between the tax rate and informal labor. As a higher formal sector productivity pulls hours towards the formal sector, a higher tax rate pushes them to the informal one. At the end the pull is stronger than the push and informal labor is reduced, similar to the previous case with varying enforcement.

4.3.4 Varying depreciation rate

Finally, we report results relative to a change in the depreciation rate of capital δ. Particularly, instead of keeping it at its calibrated value of 0.08 we now vary it between 0 and 0.15;[32] Figure 7 shows that increasing the depreciation rate of capital (and using the opposite rationale we have mentioned in the previous two cases, making the formal sector ceteris

[30] Productivity values with $\theta_F \leq 1.8$ have an associated tax rate with an imaginary component.

[31] Numerical computations suggest that setting $\rho \leq 0.62$ allows for an equilibrium to exist.

[32] As evident from Figure 7, depreciation rates above 0.15 have an associated zero tax rate.

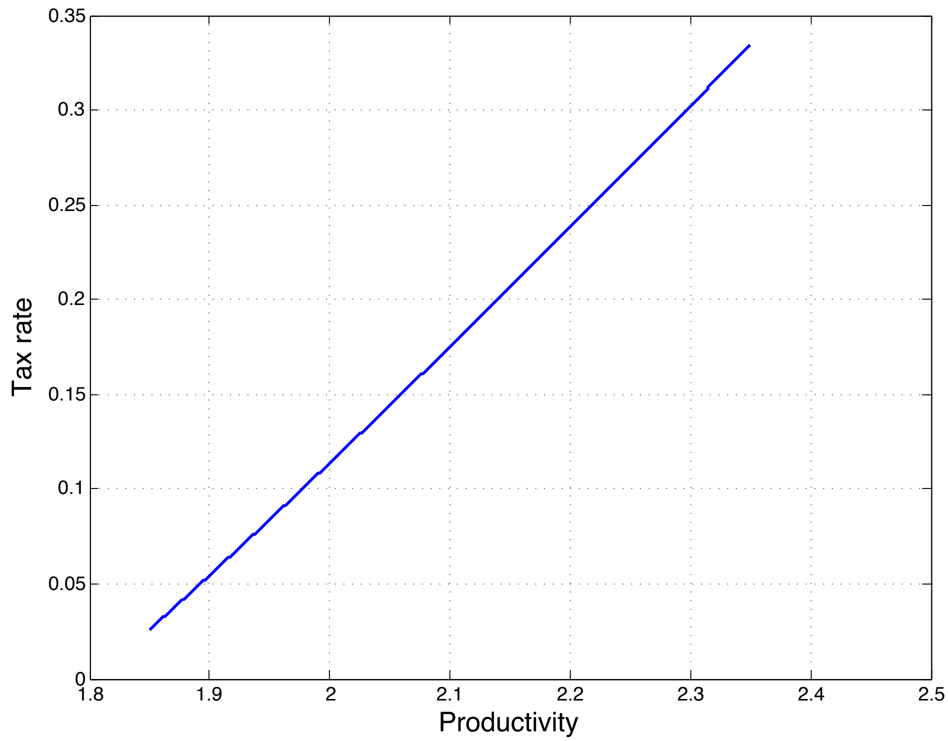

Figure 5: Revenue maximizing tax rate with variable productivity

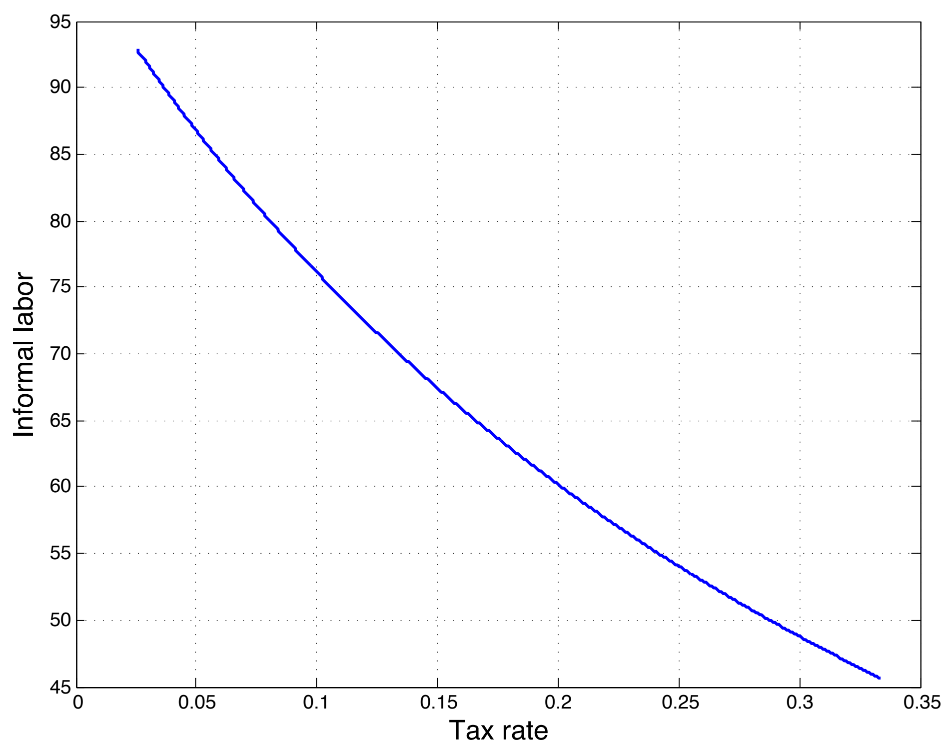

Figure 6: Informal labor and revenue maximizing tax with varying formal productivity

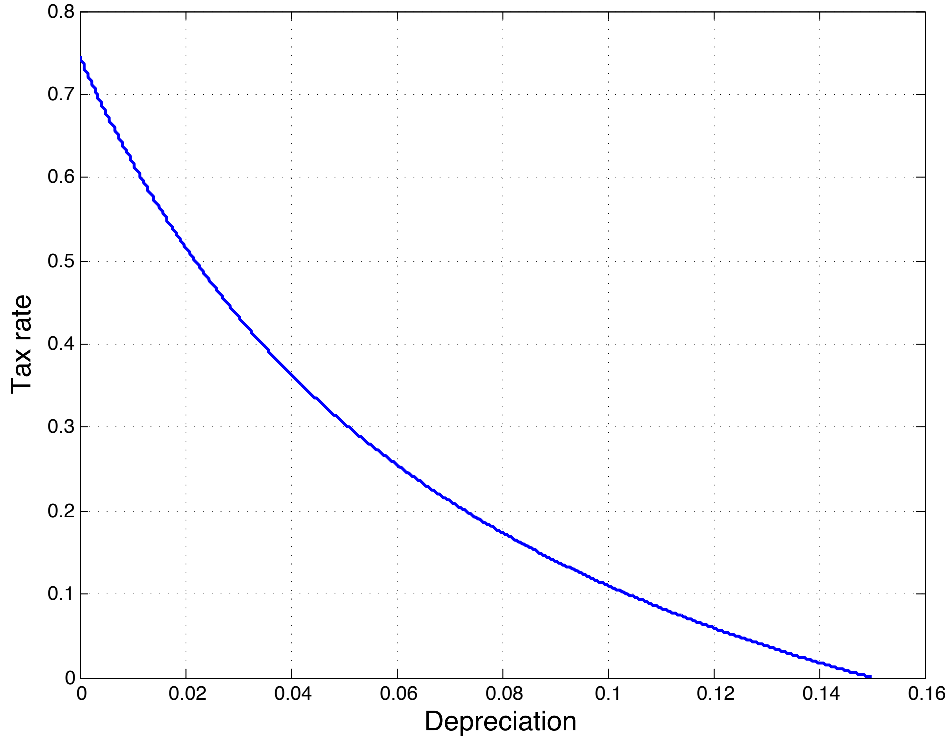

Figure 7: Revenue maximizing tax rate with variable depreciation

paribus less attractive) reduces the revenue maximizing tax rate set by the government.

As the tax rate is reduced with higher levels of depreciation, the household spends more time in the informal sector. Some intuition can be obtained by considering the extreme case where $\delta = 1$: the household's formal technology collapses to the informal one and the choice of where to allocate labor supply will depend on the particular values of θ_F and θ_I. However, as our calibration has $\theta_I > \theta_F$, one can expect that all production will take place in the informal sector.

Figure 8 illustrates the relationship between the informal labor and the tax rate with varying depreciation. The result is similar to the case with varying informal labor share. The difference is that the push effect of increasing depreciation is always stronger than the pull effect of reducing taxes and thus the household spends more labor in the informal sector. In summary, varying the depreciation rate also allows us to obtain a negative correlation between taxes and the size of the informal economy.[33]

4.4 Summing up

Model-wise, our results have the same qualitative features as those derived from the empirical analysis in Section 2. Our results show that the model can account for the properties exhibited by the data. Thus, enforcement and technology features—productivity and

[33] In McGrattan and Schmitz [15], lower maintenance and repair expenditures increase the depreciation rate. This may be another channel by which the informal sector is large in some economies and small in some others.

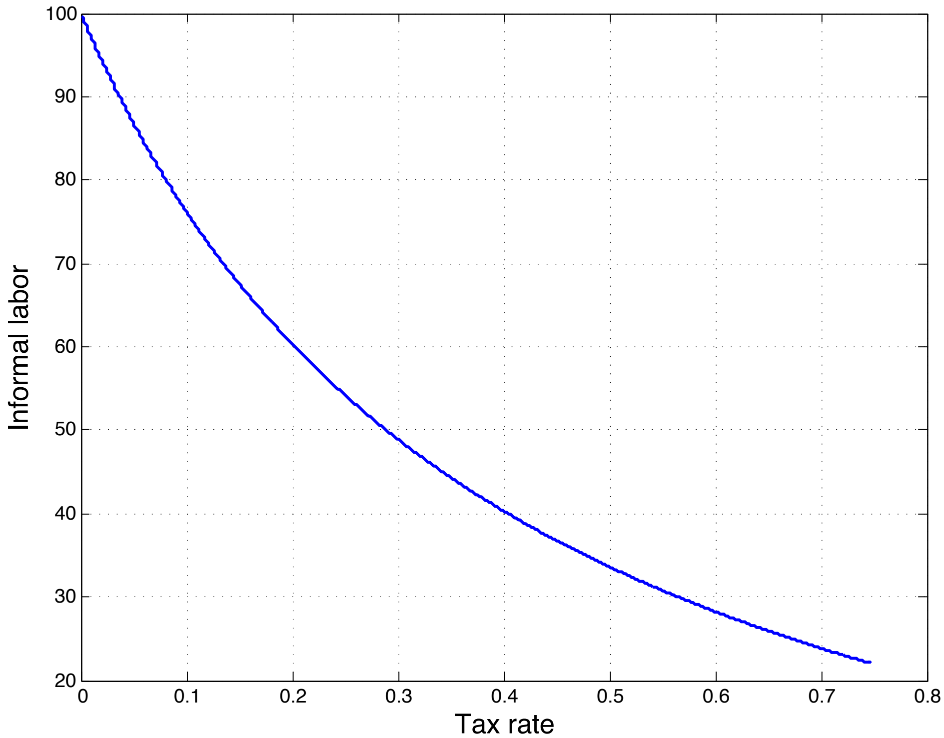

Figure 8: Informal labor and revenue maximizing tax with varying depreciation

depreciation—are likely candidates to account for the the negative relationship between taxes and informal sector size.

5 Concluding remarks

In this paper we develop a model—an extension to a two-sector growth model—to account for the negative relationship between the level of taxes and the size of the informal economy.

To this end, we endogenize the determination of taxes assuming that the government chooses the tax rate to maximize its tax revenue while taking the competitive equilibrium as given. Then, we identify several factors of our model that may affect the tax rate determined rationally by the government to maximize tax revenue, and thereby the relationship between taxes and the informal economy.

We find that (a) a higher degree of tax enforcement, (b) a higher productivity of formal sector households, and (c) a lower physical capital depreciation rate make for a negative relation between these variables. Thus, our results suggest that enforcement and technological factors are likely candidates to account for this relationship. Different than the existing literature on the relationship between taxes and informal sector size, we find that three additional factors, namely tax enforcement, productivity and depreciation also play a significant role in this relationship.

Of course, our empirical results depend critically on the data we use, and we'd be remiss if we didn't point out several subtleties of the data and how they could impact the results we obtain. First, we have no direct measure of enforceability of taxes and thus we are unsure of whether a better measurement of enforcement would change the direction of our results. Second, any criticism for measuring TFP via the Solow residual can also affect our results. Third, our measured depreciation rate comes from a steady-state relationship, yet we cannot be sure whether all countries in the sample—especially the ones with high informal sector size—are on a steady state path.

Nevertheless, we believe that our results point out several interesting aspects regarding the informal sector. A particular one which we believe deserves further research is the contribution of government policy—e.g. regulation, legal environment, or bureaucratic quality—as a determinant of productivity. Since our results suggest that increased productivity can reduce the size of the informal sector, governments interested in achieving this goal could have a major role by improving the conditions upon which firms operate.

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Technical appendix to
Tax enforcement, technology, and the informal sector

Contents

Introduction

The following results are complementary to the "Tax enforcement, technology, and the informal sector" paper.

1 Competitive equilibrium

We derive the competitive equilibrium problem in full detail. Consider the household's problem: choose sequences of consumption C, investment X, formal labor N_F, and informal labor N_I to solve

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \\ \text{s.t.} \quad & C_t + X_t = (1-\tau_t)\theta_{Ft} K_t^{\alpha} N_{Ft}^{1-\alpha} + (1-\rho(\tau_t+\tau_S))\theta_{It} N_{It}^{\gamma} && (1.1) \\ & K_{t+t} = (1-\delta)K_t + X_t && (1.2) \\ & N_{It} + N_{Ft} = T. && (1.3) \end{aligned}$$

In the above, τ denotes the formal sector tax rate, θ_F is total factor productivity (TFP) for the formal sector, K is the physical capital stock, $\alpha \in (0,1)$ is the share of capital in aggregate output, $\rho \in (0,1)$ is the level of tax enforcement by the fiscal authority, $\tau_S > 0$ is a tax surcharge imposed to informal sector output detected by the fiscal authority, θ_I is TFP for the informal sector, $\gamma \in (0,1)$ is the labor share of informal output, and $T > 0$ is the total amount of hours available per period.

For the government, we require its budget constraint to be satisfied on a period-by-period basis. Hence, government expenditure G (assumed to be wasteful) equals total tax revenue:

$$G_t = \tau_t \theta_{Ft} K_t^{\alpha} N_{Ft}^{1-\alpha} + \rho(\tau_t+\tau_S)\theta_{It} N_{It}^{\gamma}.$$

1.1 Characterization

The solution to the household problem can be derived with the Lagrangian

$$\mathscr{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{\log C_t + \lambda_t[(1-\tau_t)\theta_{Ft} K_t^{\alpha} N_{Ft}^{1-\alpha} + (1-\rho(\tau_t+\tau_S))\theta_{It} N_{It}^{\gamma} - C_t - X_t]$$
$$+ \mu_t[(1-\delta)K_t + X_t - K_{t+1}] + \eta_t(T - N_{It} - N_{Ft})\}.$$

First-order conditions are

$$\begin{aligned}
C_t &: C_t^{-1} - \lambda_t = 0 \\
N_{It} &: \lambda_t \gamma(1-\rho(\tau_t+\tau_S))\theta_{It} N_{It}^{\gamma-1} - \eta_t = 0 \\
N_{Ft} &: \lambda_t (1-\alpha)(1-\tau_t)\theta_{Ft} K_t^{\alpha} N_{Ft}^{-\alpha} - \eta_t = 0 \\
X_t &: -\lambda_t + \mu_t = 0 \\
K_{t+1} &: -\mu_t + \beta E_t[\lambda_{t+1}\alpha(1-\tau_{t+1})\theta_{F,t+1} K_{t+1}^{\alpha-1} N_{F,t+1}^{1-\alpha} + \mu_{t+1}(1-\delta)] = 0
\end{aligned}$$

together with equations (1.1)-(1.3). Combine the second and third:

$$\gamma(1-\rho(\tau_t+\tau_S))\theta_{It} N_{It}^{\gamma-1} = (1-\alpha)(1-\tau_t)\theta_{Ft} K_t^{\alpha} N_{Ft}^{-\alpha} \tag{1.4}$$

and the first and fourth in the fifth:

$$C_t^{-1} = \beta E_t C_{t+1}^{-1}[\alpha(1-\tau_{t+1})\theta_{F,t+1} K_{t+1}^{\alpha-1} N_{F,t+1}^{1-\alpha} + 1 - \delta] \tag{1.5}$$

Equations (1.1)-(1.5) characterize the household equilibrium.

1.2 Steady state

The steady state conditions corresponding to (1.1)-(1.5) are given by

$$\begin{aligned}
C + X &= (1-\tau)\theta_F K^{\alpha} N_F^{1-\alpha} + (1-\rho(\tau+\tau_S))\theta_I N_I^{\gamma} \\
X &= \delta K \\
T &= N_I + N_F \\
\gamma(1-\rho(\tau+\tau_S))\theta_I N_I^{\gamma-1} &= (1-\alpha)(1-\tau)\theta_F K^{\alpha} N_F^{-\alpha} \\
1 &= \beta[\alpha(1-\tau)\theta_F K^{\alpha-1} N_F^{1-\alpha} + 1 - \delta].
\end{aligned}$$

The first two equations solve for C and X once values for $\{K, N_I, N_F\}$ are available. Since our goal is to characterize the behavior of informal and formal sector, we only focus on the last three equations. From the fifth equation, we can rearrange to get

$$K = \left[\frac{1-\beta(1-\delta)}{\alpha\beta(1-\tau)\theta_F}\right]^{\frac{1}{\alpha-1}} N_F \tag{1.6}$$

Using this in the fourth equation yields[1]

$$N_I = \left[\frac{(1-\alpha)(1-\tau)\theta_F}{\gamma(1-\rho(\tau+\tau_S))\theta_I}\right]^{\frac{1}{\gamma-1}} \left[\frac{1-\beta(1-\delta)}{\alpha\beta(1-\tau)\theta_F}\right]^{\frac{\alpha}{(\gamma-1)(\alpha-1)}} \tag{1.7}$$

[1] Except for the minor difference in tax rates, equation (1.7) is identical to equation (7) in the paper by ?.

Finally, use the third one to get

$$N_F = T - N_I. \tag{1.8}$$

1.3 Revenue-maximizing taxes

Our goal is to find the revenue-maximizing tax rate conditional on the values of enforcement ρ, formal-sector TFP θ_F, and physical capital depreciation rate δ. Steady-state tax revenue is given by

$$\mathcal{R} = \tau\theta_F K^\alpha N_F^{1-\alpha} + \rho(\tau + \tau_S)\theta_I N_I^\gamma.$$

The revenue-maximizing tax rate is the one that satisfies $\partial\mathcal{R}(\tau)/\partial(\tau) = 0$, where

$$\frac{\partial\mathcal{R}}{\partial\tau} = \theta_F \left\{ \tau \left[(1-\alpha)\left(\frac{K}{N_F}\right)^\alpha \frac{\partial N_F}{\partial\tau} + \alpha\left(\frac{K}{N_F}\right)^{\alpha-1} \frac{\partial K}{\partial\tau} \right] + K^\alpha N_F^{1-\alpha} \right\} + \rho\theta_I N_I^\gamma \left[\frac{\gamma(\tau+\tau_S)}{N_I}\frac{\partial N_I}{\partial\tau} + 1 \right].$$

Closed-form expressions for $\{K, N_I, N_F\}$ follow equations (1.6)-(1.8); from these equations we can easily get the required derivatives as[2]

$$\frac{\partial N_I}{\partial\tau} = \frac{N_I}{\gamma-1}\left[\frac{\rho}{1-\rho(\tau+\tau_S)} + \frac{1}{(\alpha-1)(1-\tau)}\right] \tag{1.9}$$

$$\frac{\partial N_F}{\partial\tau} = -\frac{\partial N_I}{\partial\tau} \tag{1.10}$$

$$\frac{\partial K}{\partial\tau} = K\left[\frac{1}{N_F}\frac{\partial N_F}{\partial\tau} + \frac{1}{(\alpha-1)(1-\tau)}\right]. \tag{1.11}$$

2 Matlab files

Matlab and data files can be downloaded from http://www.macalester.edu/~msolisga.[3]

`TAXRATE.M` This file replicates the typical positive relationship between taxes and informal sector size assuming that taxes are exogenously given. Executing `TAXRATE.M` generates two graphs, where the first one is used in the paper. The first graph shows the size of the informal sector as a function of the formal tax rate, while the second one graph shows tax revenue as a function of the tax rate.

`REVENUE.M` Auxiliary file for the `ENFORCEMENT.M`, `PRODUCTIVITY.M`, and `DEPRECIATION.M` scripts. It finds the optimal tax rate by solving $\partial\mathcal{R}(\tau)/\partial(\tau) = 0$.

[2] See Appendix A for additional details on these calculations.

[3] In order to run the files listed below, two additional Matlab routines are required. The first one, `NEWTON.M` implements a simple Newton-Raphson procedure in order to find the zero of a function (in this case, $\partial\mathcal{R}(\tau)/\partial(\tau) = 0$). The second one, `JACOBIAN.M`, is an auxiliary file used by `NEWTON.M`.

ENFORCEMENT.M This script finds the revenue-maximizing tax rate as the degree of enforcement ρ varies within user-specified bounds. Executing ENFORCEMENT.M generates four graphs; the first two are used in the paper. The first one shows the revenue-maximizing tax rate as a function of enforcement. The second one shows the size of the informal sector as a function of the formal tax rate, while the third one does so as a function of enforcement. Finally, the fourth graph shows the tax revenue as a function of enforcement.

PRODUCTIVITY.M This script replicates ENFORCEMENT.M but varies the formal household productivity θ_F instead.

DEPRECIATION.M This script replicates ENFORCEMENT.M but varies the physical capital depreciation rate δ instead.

A Additional calculations

From Section 1.2,

$$N_I = \left[\frac{(1-\alpha)(1-\tau)\theta_F}{\gamma(1-\rho(\tau+\tau_S))\theta_I}\right]^{\frac{1}{\gamma-1}} \left[\frac{1-\beta(1-\delta)}{\alpha\beta(1-\tau)\theta_F}\right]^{\frac{\alpha}{(\gamma-1)(\alpha-1)}}$$

Rewrite as

$$N_I = \phi_I (1-\tau)^{-\frac{1}{(\gamma-1)(\alpha-1)}} (1-\rho(\tau+\tau_S))^{\frac{1}{\gamma-1}} \tag{A.1}$$

where

$$\phi_I \equiv \left[\frac{(1-\alpha)\theta_F}{\gamma\theta_I}\right]^{\frac{1}{\gamma-1}} \left[\frac{1-\beta(1-\delta)}{\alpha\beta\theta_F}\right]^{\frac{\alpha}{(\gamma-1)(\alpha-1)}}$$

The derivative of equation (A.1) with respect to τ is

$$\begin{aligned}
\frac{\partial N_I}{\partial \tau} &= \phi_I \left\{ \left(\frac{\rho}{\gamma-1}\right)(1-\tau)^{-\frac{1}{(\gamma-1)(\alpha-1)}} (1-\rho(\tau+\tau_S))^{\frac{1}{\gamma-1}-1} \right. \\
&\qquad \left. + \left[\frac{1}{(\alpha-1)(1-\tau)}\right](1-\rho(\tau+\tau_S))^{\frac{1}{\gamma-1}} (1-\tau)^{-\frac{1}{(\gamma-1)(\alpha-1)}-1} \right\} \\
&= \frac{\phi_I (1-\tau)^{-\frac{1}{(\gamma-1)(\alpha-1)}} (1-\rho(\tau+\tau_S))^{\frac{1}{\gamma-1}}}{\gamma-1} \left[\frac{\rho}{1-\rho(\tau+\tau_S)} + \frac{1}{(\alpha-1)(1-\tau)}\right] \\
&= \frac{N_I}{\gamma-1}\left[\frac{\rho}{1-\rho(\tau+\tau_S)} + \frac{1}{(\alpha-1)(1-\tau)}\right]
\end{aligned}$$

which is equation (1.9) in Section 1.3. Also, $N_F = T - N_I$, so that

$$\frac{\partial N_F}{\partial \tau} = -\frac{\partial N_I}{\partial \tau}.$$

Finally,

$$K = \left[\frac{1-\beta(1-\delta)}{\alpha\beta(1-\tau)\theta_F}\right]^{\frac{1}{\alpha-1}} N_F.$$

Rewrite as

$$K = \phi_K (1-\tau)^{-\frac{1}{\alpha-1}} N_F \tag{A.2}$$

where

$$\phi_K \equiv \left[\frac{1-\beta(1-\delta)}{\alpha\beta\theta_F}\right]^{\frac{1}{\alpha-1}}$$

The derivative of equation (A.2) with respect to τ is

$$\begin{aligned}
\frac{\partial K}{\partial \tau} &= \phi_K \left[(1-\tau)^{-\frac{1}{\alpha-1}} \frac{\partial N_F}{\partial \tau} + \left(\frac{1}{\alpha-1}\right)(1-\tau)^{-\frac{1}{\alpha-1}-1} N_F\right] \\
&= \phi_K (1-\tau)^{-\frac{1}{\alpha-1}} N_F \left[\frac{1}{N_F}\frac{\partial N_F}{\partial \tau} + \frac{1}{(\alpha-1)(1-\tau)}\right] \\
&= K \left[\frac{1}{N_F}\frac{\partial N_F}{\partial \tau} + \frac{1}{(\alpha-1)(1-\tau)}\right]
\end{aligned}$$

which is equation (1.11) in Section 1.3.