Can Sustained Economic Growth and Declining Population Coexist? Barro-Becker Children Meet Lucas[*]

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August 24, 2010

Abstract

Many developed countries have been experiencing sub-replacement fertility. This leads to worries over the sustainability of economic growth in these countries. Given this concern, we ask the following questions: Is there a force out there that would allow economic growth and declining population to coexist? Is there a mechanism that could push fertility back up? We argue that returns to human capital in production provide the key to understanding this relation. Our theoretical framework predicts that, when the degree of increasing returns to human capital in traditional production technologies falls, advanced economies switch their productive efforts from labor-oriented technologies that require a constant creation of young workforce toward human capital-oriented technologies that support an ageing population. We call this shift the "endogenous efficiency-augmenting mechanism". This suggests that sustained economic growth and a declining population can coexist in the long run. Finally, we compare our model against the data and find: (*i*) The degree of increasing returns to human capital has been falling over time throughout the world along with population growth rates. (*ii*) Increasing returns to human capital and population growth rates are positively correlated. (*iii*) Predictions of our model are consistent with what the data reveal.

JEL codes: J11, J24, O41.

Keywords: Economic growth; declining population; endogenous fertility; increasing returns.

[*]We thank Larry Jones for helpful comments and suggestions. We also thank Steve Durlauf, James Heckman, and Rafael Lopes de Melo for their continuous support. The views expressed here are of our own and do not necessarily reflect those of the Central Bank of the Republic of Turkey. All errors are ours.

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1 Introduction

A majority of the developed countries are expected to face declining populations in the foreseeable future if their fertility rates stay below the replacement rates. Japan is the most prominent example. As a result of five consecutive decades of low birth rates and almost no net immigration, its population fell by 0.01 percent in 2005, followed by zero growth in 2006 and 2007, and a 0.06 percent decline in 2008. Recent population projections of the Ministry of Internal Affairs and Communications in Japan suggest that Japanese population, which is currently around 127 million, will decline steadily during the first half of the 21st century to reach 95 million—close to its 1960 levels—in 2050. Similar concerns are raised for many European and Asian nations. The United States has relatively high fertility, but still way below its historical standards.

Among the most significant issues is the impact of declining population on economic growth. In this paper, we ask the following questions: Is there a force out there that would allow economic growth and declining population to coexist? Is there a mechanism that could push fertility back up? The literature on fertility and economic growth argues that, *along the development path*, parents have fewer children each with higher quality. As a result, fertility declines and the stock of human capital grows, which lead to sustained economic growth in per capita terms. This is the main mechanism suggested by many papers including Becker, Murphy, and Tamura (1990) and Galor and Weil (1999, 2000). It explains what happens at the early stages of development quite well.[1] Our research is motivated by the observation that there is no serious theory linking fertility, human capital accumulation, and economic growth once we get to the modern advanced economy with below replacement fertility. There are serious calls for such a contribution (see Galor (2008) and Murphy (2009)). Unlike the majority of papers in the fertility and growth literature, we do *not* attempt to model transition from underdevelopment to development. Our goal is to construct a model of fertility and growth in advanced societies rather than to investigate what happens during the process of development.

This paper argues that the degree of increasing returns to human capital in traditional production technologies determines the nature of the long-term relationship between economic growth and population growth in advanced countries. By traditional technologies, we mean labor-oriented production processes prevalent in traditional manufacturing and services sectors. These sectors have benefited largely from increasing returns to human capital generated through agglomeration economies and the formation of cities throughout the second half of the twentieth century (see Moretti (2004a), Moretti (2004b)).

Our theoretical structure combines the models developed by Becker and Lewis (1973), Becker and Barro (1988), Becker, Murphy, and Tamura (1990), and Lucas (1988). Like Becker and Lewis

[1] See, for example, Galor and Weil (1996), Galor and Moav (2002), Hazan and Berdugo (2002), Lagerloef (2003), Fernandez-Villaverde (2004), Moav (2005), and Gould, Moav, and Simhon (2008) for different perspectives on the determinants of this demographic transition. See Galor (2005) for an excellent survey.

(1973) and Becker, Murphy, and Tamura (1990), quantity-quality tradeoff at the aggregate level is an important element in our analysis. Unlike these papers, we generate this tradeoff in a novel way: the goods cost of child rearing is an increasing function of the aggregate human capital. This assumption communicates the idea that human capital is a cost element as well as a productivity element for the society; that is, part of the human capital stock is accumulated for the purpose of raising higher quality children. When the population starts declining, this mechanism works in the opposite direction to produce extra resources for the society. We explain this "efficiency-augmenting mechanism" in greater detail in Section 2. Like Becker and Barro (1988) and Becker, Murphy, and Tamura (1990), our model is based on a dynastic utility structure. Like Becker, Murphy, and Tamura (1990), we use this dynasty structure to construct a growth model based on aggregate human capital. Unlike all other papers in the literature, we insert this structure into an endogenous growth model—a là Lucas (1988)—with increasing returns to aggregate human capital in production. We work out a particular balanced growth path and perform our analysis across steady state solutions. Our model produces two main predictions. As the degree of increasing returns to human capital in traditional production technologies falls,

(1) the population growth rates decline steadily, whereas the growth rates of per capita consumption display a U-shaped pattern; and

(2) negative population growth and positive growth in per capita consumption can coexist.

The first result says that when the increasing returns to human capital in traditional production technologies are large, both the per capita consumption growth rate and population growth rate are fairly high. As the increasing returns move down, the growth rates of per capita consumption follow a U-shaped pattern. It is always positive. However, the population growth rates fall steadily and can record negative numbers after reaching a threshold. The second result follows immediately.

The **first** question is: Is there any empirical evidence that increasing returns to human capital have been falling throughout the world and are positively correlated with population growth rates? We perform an empirical analysis in Section 3 and our answer is yes. Using educational attainment data from Barro and Lee (2010), and for a sample of the biggest 50 countries in terms of per capita consumption, we show that population growth rates and the returns to human capital are positively correlated. Moreover, we provide time-series evidence that the degree of increasing returns to human capital has been falling over time. These results are in line with the literature arguing that population growth and the returns to human capital are positively related (Glaeser, Kallal, Scheinkman, and Shleifer (1992), Kremer (1993), and Glaeser (1999)). Our estimates also show that the growth rates of per capita consumption follow a U-shaped pattern as the degree of increasing returns to human capital falls. These estimates are robust since we carry out empirical analysis in various dimensions including the time-series, panel-data, and cross-sectional perspectives, all of which yield similar results. These analyses provide empirical confirmation that our theoretical results are sensible.

The motivation behind modeling the decline in the degree of increasing returns to human capital in traditional production technologies comes from the literature investigating the population structure in cities. Population in large cities starts going down after reaching a peak point (Williamson (1965), Hansen (1990), and Henderson (2003)). This de-concentration happens for two reasons: (*i*) the economy spreads knowledge to hinterland areas, or (*ii*) cities become overcongested for the inhabitants. This regularity documented in empirical work including El-Shakhs (1972), Alonso (1980), Wheaton and Shishido (1981), Junius (1999), and Davis and Henderson (2003). Our theory is the macroeconomic counterpart of this argument.

The **second** issue is about how the model works. Traditional technologies are characterized by labor-dependency in production. When the increasing returns to human capital are large, the role of human capital on the aggregate productive capacity is so highly significant that the society can afford the cost of human capital used in producing children. Children will start working when they grow up. In traditional production technologies, labor is complementary to human capital. Thus, greater human capital stock generates greater work force and the economy grows as a whole. As the degree of increasing returns to human capital in traditional production technologies starts decreasing, the positive gap between the returns from human capital and the cost of producing it decreases. This makes the growth rates of human capital (and, therefore, the growth rates of per capita consumption) decline. The complementarity between human capital and labor effort drives the population growth rates down. The endogenous efficiency-augmenting mechanism leads to the emergence of alternative labor-saving production methods. These methods liberate the gains in productivity from the counterbalancing effects of population. After a certain threshold, the population growth rates read negative numbers. Eventually, we reach a point where the degree of increasing returns are low, population declines, and per capita consumption (and human capital) grows. This mechanism paves the way for accelerated human capital accumulation and the emergence of the modern state of sustained economic growth. Kosai, Saito, and Yashiro (1998) argue that such a mechanism should necessarily work for the potential coexistence of sustained economic growth and declining population without explicitly describing the mechanism. Our paper proposes one such mechanism.

What justifies the emergence of such a mechanism? Think of the banking sector. It used to be one of the most labor-intensive sectors. Parallel to the rapid development of the internet technology, the sector now trends toward to a concept called branchless banking. Another example is the transformation that the technologies producing household durables and other high-tech commodities have undergone. In Japan, human involvement in production and quality-control stages in most plants are literally zero. It is possible to extend the list of examples. This transformation leads to the emergence of alternative production technologies. In other words, the continuous increase in the productive capacities of these alternative technologies supports lower population growth rates. Our model argues that all these developments are endogenous.

The **third**, and the last, question is: why is this progress? We contribute the literature in two dimensions. (*i*) We show that the decline in the degree of increasing returns to human capital drives population growth rates down to negative levels in the developed countries. This result is important because it reconciles two competing views. On the one hand, the traditional growth literature says that a larger population may hamper economic growth because of the diminishing returns from more intensive use of land and other natural resources (Krugman (1991), Ciccone and Hall (1996)). On the other hand, the urban economics argues that population growth and economic growth are positively related since larger populations encourage greater specialization and increased investments in knowledge (Kremer (1993) and Glaeser (1999)). These two views have long been argued as being rival (see Becker, Glaeser, and Murphy (1999) for an explanation why these views are believed to be rival). Our paper shows that these two are related issues and they can can co-habit in a single model. (*ii*) We propose an endogenous growth model which shows that a declining population and sustained economic growth in per capita terms can coexist. This result suggests that a declining population is not such a big problem when the individual well-being is concerned. Note that, in a standard Barro-Becker model with exogenous growth (see Section 9.2.2 in Barro and Sala-i Martin (2003) for an example), it is possible to find a parameter configuration that gives a negative relationship between population growth and per capita consumption growth. Our analysis is much richer in the sense that we describe a mechanism that produces an endogenous transition from a positive relationship between population growth and per capita consumption growth to a negative one.

The paper is organized as follows. Section 2 describes the theoretical model and characterizes the steady state solution. Section 3 shows that our numerical results are coherent and consistent with the empirical evidence. Section 4 concludes.

2 The Environment

2.1 The Dynastic Utility

We build our analysis on the Barro-Becker model of dynastic altruism (see Becker and Barro (1988)). Our starting point is a standard two-period overlapping generations setting. The utility of an adult is an additively separable function of his own consumption and the utility of each child weighted by the number of children and the degree of altruism toward each child. After recursively substituting the utility of adult in the utility of children, we obtain a dynastic utility representation similar to the one in the Barro-Becker model. We skip the derivation of the resulting dynastic utility function and jump ahead into the problem.

For simplicity, we formulate our model in continuous time (see Section 9.2.2 in Barro and Sala-i Martin (2003) for a detailed explanation why a continuous-time overlapping generations formulation is useful when one studies aggregate variables rather than family-level variables). This leads us to the following utility function for the dynastic head (or the time-zero agent):

$$U_0 = \int_0^\infty e^{-\alpha t} N(t)^{\sigma-\epsilon} \frac{C(t)^{1-\sigma}}{1-\sigma} dt \tag{2.1}$$

where $N(t)$ is population, $C(t)$ is aggregate consumption, $\epsilon \in (0,1)$ is the constant elasticity of altruism per child as their number increases, $\alpha \in (0,1)$ is the rate of "pure" altruism, and $\sigma > 0$ is the inverse of the elasticity of substitution.[2]

To comply with the standard assumptions in the fertility literature and to ensure that the dynastic utility function is increasing and concave in $C(t)$ and $N(t)$, we impose the following restrictions on the parameters.

Assumption 2.1. $0 < \epsilon < \sigma < 1$.

Let $u(C(t), N(t)) = N(t)^{\sigma-\epsilon} \frac{C(t)^{1-\sigma}}{1-\sigma}$ denote the period utility function. The first inequality, $\epsilon > 0$, ensures joint concavity, i.e., $u_{CC} < 0$, $u_{NN} < 0$, and $u_{CN}u_{NC} - u_{CC}u_{NN} < 0$. The second inequality, $\sigma < 1$, implies $u > 0$ and $u_C > 0$. Finally, $u_N > 0$ is ensured by $\epsilon < \sigma$. These parametric restrictions imply that aggregate consumption and population are complements—the cross partial of the period utility is positive—in dynastic utility. In other words, the period utility is less than log utility, which implies a low curvature or a high elasticity of substitution. This is the usual assumption in the Barro-Becker tradition. We maintain the complementarity hypothesis throughout the paper.

2.2 Production, Resources, and Demographics

Our formulation of the technology and the resource constraint are closer to the Uzawa-Lucas endogenous growth tradition. Following Lucas (1988), we assume that human capital evolves over time according to the law of motion

$$\dot{h}(t) = \gamma h(t)\big[1 - u(t)\big], \tag{2.2}$$

where $\gamma \in (0,1)$ is the effectiveness of investment in human capital, $\dot{h}(t)$ denote the growth of human capital, and $[1 - u(t)] \in [0,1]$ is the fraction of total effort devoted to acquiring more human capital. The rest of the effort, $u(t)$, is devoted to producing the consumption good. Production in this economy depends on the levels of human capital and labor inputs.

[2] This is equivalent to the setup in Jones and Schoonbroodt (2010) and delivers simple analytical comparative statics results across balanced growth paths.

Accordingly, the resource constraint is

$$\left[u(t)N(t)\right]^{1-\theta}h(t)^{\theta+\eta} = C(t) + \beta h(t)\dot{N}(t), \tag{2.3}$$

where $\theta \in (0,1)$, $\eta \in (0,1)$, and $\beta > 0$. η reflects the degree of increasing returns to human capital. Notice that, unlike Lucas (1988), we do not distinguish between private and aggregate variables for human capital, since we are not interested in the private solution. The increasing returns parameter η is the key to our analysis. We call the production technology on the left-hand side of the resource constraint (2.3) *traditional*; that is, labor effort and the human capital stock enter the Cobb-Douglas production technology as complementary inputs.[3]

The traditional assumption in the literature is that, other than the time cost, there is a "goods cost" of child rearing and it is only a function of the number of children; that is, in our language, the literature formulates it as $\beta\dot{N}(t)$, where β converts the number of children into consumption units (Becker and Barro (1988), Becker, Murphy, and Tamura (1990)). We believe that this convention is inadequate since the goods cost of child rearing is actually a combination of the number of children and the human capital stock in the economy. In other words, the goods cost of child-rearing is $\beta h(t)\dot{N}(t)$ in our model, where β converts this combination into the consumption units. In terms of the planning problem, this means that when the planner chooses the optimal path for the human capital accumulation, he should take into account the fact that human capital is actually used in child rearing. This is an important feature of our analysis.

Notice that $\beta h(t)\dot{N}(t)$ is a positive number—therefore, a cost element—when the population growth rate is positive, whereas it turns negative—therefore, becomes an element of production—when the population growth rate is negative. Transition of the sign of $\beta h(t)\dot{N}(t)$ from positive to negative is the endogenous efficiency-augmenting mechanism that we discuss in this paper. When the degree of increasing returns to human capital (η) in traditional technologies decline, less children are produced since the productivity of the labor input starts falling short of the cost of raising them. After some point, the population starts declining. Notice that when $\dot{N}(t) < 0$, additional units of the consumption good are produced in an alternative environment. We interpret this shift as the transition from traditional (labor-oriented) production techniques into the ones that are more compatible with an aging population—such as branchless banking, humanless production lines, humanless quality control, etc. In a realistic case, these two production processes coexist, which means that population growth and per capita consumption growth coexist.

Such a formulation is consistent with what the basic quantity-quality theory of children says. Becker and Lewis (1973) and Becker and Tomes (1976) argue that the cross partial of the cost function, $\beta h(t)\dot{N}(t)$, must have a positive sign. In other words, the cost of an additional child,

[3] Including physical capital into this technology does not change the main predictions of our model. Hence, we drop physical capital for algebraic simplicity. We include physical capital in Section 3.2, where we report our estimates.

holding the quality input fixed, is greater the higher the number of children is. Similarly, the cost of a unit increase in quality, holding the quantity fixed, is greater the higher the number of children is. $\beta > 0$ ensures that the sign of this cross partial is positive.

Notice the peculiar nature of the demographics in our model. In the literature, the standard law of motion for population is $n(t) = b(t) - d(t) = \dot{N}(t)/N(t)$, where $b(t)$ is the birth rate, $d(t)$ is the death rate, and $n(t)$ is the resulting net rate of change in population at time t. We do not explicitly model $b(t)$ and $d(t)$ in our model and focus on $n(t)$ only, which will become a constant, κ_N, along the balanced growth path. In other words, we abstract from the considerations such as the size of each generation or the number of children produced in each period. In such a setup, a negative (positive) net population growth rate means that death rate is higher (lower) than the birth rate in the economy. We do this simplification since we are only interested in whether the steady state population growth rate κ_N is positive or negative. Similar to Lucas (1988), our interpretation of $h(t)$ is that it is the aggregate know-how in an economy rather than being generation-specific human capital. This is the main reason why we do not need to explicitly model the birth/death process. We focus on the interaction between the properties of the aggregate production process and the aggregate population dynamics, which does not require explicit birth and death rates.

Two key features of this discussion should perhaps be reemphasized. First, there is increasing returns to human capital. Second, the goods cost of child rearing is a function of both human capital and the number of children, rather than being a function of the number of children only. The former determines whether a declining population is optimal and the latter ensures that there is a particular solution to our model (i.e., a particular balanced growth path) with properties we are specifically interested in.

2.3 The Planner's Problem

The social planner chooses the time paths of the dynasty level variables (see Alvarez (1999) for theoretical justification). The current-value Hamiltonian is

$$\mathcal{H}(N, h, \mu, \lambda, c, u, t) = N^{\sigma-\epsilon}\frac{C^{1-\sigma}}{1-\sigma} + \mu\left\{\frac{1}{\beta}\left[(uN)^{1-\theta}h^{\theta+\eta-1} - Ch^{-1}\right]\right\} + \lambda\big[\gamma h\,(1-u)\big] \tag{2.4}$$

where $\mu(t)$ and $\lambda(t)$ are the shadow prices used to value increments to population and human capital, respectively. There are two control variables: the aggregate consumption, $C(t)$, and the time devoted to producing the consumption good, $u(t)$. The first-order conditions are

$$N^{\sigma-\epsilon}C^{-\sigma} = \frac{\mu}{\beta h} \tag{2.5}$$

and

$$\frac{\mu}{\beta}(1-\theta)u^{-\theta}N^{1-\theta}h^{\theta+\eta-2} = \lambda\gamma. \tag{2.6}$$

Equation (2.5) states that, on the margin, the consumption good must be equally valuable across its two uses: consumption and producing children. Similarly, Equation (2.6) states that time, the other control variable, must be equally valuable in its two uses: production of human capital and production of children.

The shadow prices evolve according to

$$\dot{\mu} = \alpha\mu - \frac{\sigma-\epsilon}{1-\sigma}N^{\sigma-\epsilon-1}C^{1-\sigma} - \frac{\mu}{\beta}(1-\theta)u^{1-\theta}N^{-\theta}h^{\theta+\eta-1} \tag{2.7}$$

and

$$\dot{\lambda} = \alpha\lambda - \frac{\mu}{\beta}\Big[(\theta+\eta-1)(uN)^{1-\theta}h^{\theta+\eta-2} + Ch^{-2}\Big] - \lambda\gamma(1-u). \tag{2.8}$$

Thus, Equations (2.2) and (2.3) and (2.5)-(2.8), together with the transversality conditions

$$\lim_{t\to\infty} e^{-\alpha t}\mu(t)N(t) = 0 \tag{2.9}$$

and

$$\lim_{t\to\infty} e^{-\alpha t}\lambda(t)h(t) = 0, \tag{2.10}$$

implicitly describe the optimal evolution of $N(t)$ and $h(t)$ from any initial mix of these two state variables.

2.4 Characterizing the Balanced Growth Path

We start by guessing that there is a particular solution (a particular balanced growth path) in which u is constant. Let this constant be $\tilde{u}$. In fact, the same constant $\tilde{u}$ will work no matter what h we have. To be concrete, we rewrite Equation (2.2) to get:

$$\kappa_h = \frac{\dot{h}(t)}{h(t)} = \gamma(1-\tilde{u}) \tag{2.11}$$

where κ_h is the steady state growth rate of the aggregate human capital stock. In other words, $\tilde{u}$ is independent of the level of h. If there exist such a $\tilde{u}$, then (2.11) holds. The discussion in the rest of this section proves, in a rather informal way, that such a solution exists.

Plugging (2.5) and the resource constraint (2.3) into (2.7) yields the following formula:

$$\frac{C(t)}{N(t)h(t)} = \beta \left[\frac{\alpha - (1-\theta)\kappa_N - \kappa_\mu}{\frac{\sigma-\epsilon}{1-\sigma} + (1-\theta)} \right] \tag{2.12}$$

where $\kappa_N = \dot{N}(t)/N(t)$ and $\kappa_\mu = \dot{\mu}(t)/\mu(t)$ are constants by definition of a balanced growth path. Equation (2.12) implies that $C(t)/(N(t)h(t))$ is constant or, differentiating, that

$$\kappa_C = \kappa_N + \kappa_h \tag{2.13}$$

where $\kappa_C = \dot{C}(t)/C(t)$. This expression states that, along the steady-state growth path, aggregate consumption growth will be equal to the sum of the growth rates of population and human capital stock. Obviously, $\kappa^* = \kappa_C - \kappa_N$ is the growth rate of per capita consumption. Therefore, Equation (2.13) says that per capita consumption growth must be equal to the growth rate of human capital. The striking result is that nonnegative values for population growth is not necessary for sustainable economic growth. Becker, Glaeser, and Murphy (1999) sketch out a similar idea without providing a formal model. This paper complements their effort by introducing a fully specified growth model. Next we examine what determines whether κ_N is negative or positive.

The resource constraint (2.3) can be rewritten as

$$\tilde{u}^{1-\theta} N(t)^{-\theta} h(t)^{\theta+\eta-1} = \frac{C(t)}{N(t)h(t)} + \beta\kappa_N. \tag{2.14}$$

The right-hand side is a constant by (2.12). It means that the left-hand side must also be a constant. Differentiating (2.14) gives us the following key result:

$$\kappa_N = \frac{\theta+\eta-1}{\theta}\kappa_h. \tag{2.15}$$

We summarize what this expression communicates in the following proposition.

Proposition 1. *At an interior solution for u, κ_h is positive and*

(i) $\kappa_N > 0$, *if* $\theta + \eta > 1$;

(ii) $\kappa_N = 0$, *if* $\theta + \eta = 1$, *and;*

(iii) $\kappa_N < 0$, *if* $\theta + \eta < 1$.

In words, when the degree of increasing returns to human capital in traditional production processes are strong enough to yield $\theta + \eta > 1$, population and per capita consumption grow together. When the opposite is true, population growth rates are negative and the growth rates of per capita consumption are positive.

Proposition 1 states that economic growth can be sustained in modern economies regardless of the sign of the growth rate of population. When increasing returns are strong enough (i.e., $\theta + \eta > 1$), returns to a growing population is larger than the total cost of child rearing. Therefore, population grows and reinforces the economic growth. When the opposite is true, a steady decline in population combined with growing human capital stimulates an efficiency-augmenting mechanism. This is more easily seen through the aggregate resource constraint. When κ_N is negative, the cost of child rearing, $\beta h(t)\dot{N}(t)$, reads a negative number. The intuition is the following. An increasing scarcity of labor would stimulate more efficient utilization of resources, highlighting "gains in efficiency". In other words, an endogenously induced mechanism counteracts the decline in population.

To complete the characterization of the steady-state, we differentiate the first-order conditions (2.5) and (2.6), and use (2.15), to get

$$\kappa_\mu = (\sigma - \epsilon)\kappa_N - \sigma\kappa_C + \kappa_h \tag{2.16}$$

and

$$\kappa_\lambda - \kappa_\mu = \kappa_N - \kappa_h, \tag{2.17}$$

where $\kappa_\lambda = \dot{\lambda}(t)/\lambda(t)$. One more equation is needed to close the system with six linearly independent equations in six unknowns. Combining (2.7) and (2.8), we obtain

$$\kappa_\lambda = (\alpha - \gamma) + \frac{\gamma\tilde{u}}{1-\theta}\left[1 - 2\theta - \eta + \frac{\frac{\sigma-\epsilon}{1-\sigma}\kappa_N + (1-\theta)\kappa_N}{\alpha + \frac{\sigma-\epsilon}{1-\sigma}\kappa_N - \kappa_\mu}\right]. \tag{2.18}$$

A full characterization of the balanced growth path requires that the transversality conditions (2.9) and (2.10) jointly hold. The first one is satisfied if $\kappa_\mu + \kappa_N < \alpha$. Similarly, the second one is satisfied if $\kappa_\lambda + \kappa_h < \alpha$. These are standard conditions and they should be paid attention when parameterizing the model.

2.5 Solution

Notice that each and every variable in the system can be rewritten as a function of $\tilde{u}$ only, although there is no analytical solution to our model. The easiest way to solve for this system is to carry out a simple fixed point analysis. We take two equations containing κ_λ and write them as functions of $\tilde{u}$ only. This yields two equations in two unknowns.

From (2.17), we obtain

$$\kappa_\lambda^\ell = \underbrace{\frac{(\theta+\eta-1)\gamma}{\theta}(1-\tilde{u})}_{\kappa_N} - \underbrace{\gamma(1-\tilde{u})}_{\kappa_h} + \underbrace{(1-\sigma)\gamma(1-\tilde{u}) - \frac{(\theta+\eta-1)\gamma\epsilon}{\theta}(1-\tilde{u})}_{\kappa_\mu} \tag{2.19}$$

where we let κ_λ^ℓ denote the left-hand side of the fixed point equation. Collecting the terms with $\gamma(1-\tilde{u})$ and rearranging Equation (2.19), we get

$$\kappa_\lambda^\ell = \left[\frac{(\theta+\eta-1)}{\theta}(1-\epsilon) - \sigma\right]\gamma(1-\tilde{u}). \tag{2.20}$$

On the right-hand side, we have the expression coming from Equation (2.18). After substituting κ_N and κ_μ into (2.18), the right-hand side becomes

$$\kappa_\lambda^r = (\alpha-\gamma) + \frac{\gamma\tilde{u}}{1-\theta}\left[1-2\theta-\eta+\frac{\frac{\sigma-\epsilon}{1-\sigma}\kappa_N + (1-\theta)\kappa_N}{\alpha+\frac{\sigma-\epsilon}{1-\sigma}\kappa_N - \kappa_\mu}\right]. \tag{2.21}$$

Thus, $\tilde{u}$ that solves our system can be described as follows:

$$\tilde{u} = \left\{u \in (0,1) \middle| \kappa_\lambda^\ell(u) = \kappa_\lambda^r(u)\right\}. \tag{2.22}$$

To sketch out the characteristics of the solution more concretely and to establish a unique solution to our model, we impose the following additional restriction:

Assumption 2.2. $\sigma + \epsilon > 1$.

Proposition 2. *Under Assumption 2 and given $\theta, \eta, \epsilon \in (0,1)$, κ_λ^ℓ is monotonically increasing in u.*

Proof: For κ_λ^ℓ to increase monotonically in u, it has to be the case from (2.20) that $\sigma\theta > (1-\epsilon)(\theta+\eta-1)$. Rearranging this inequality yields $\theta(\sigma+\epsilon-1) > (\eta-1)(1-\epsilon)$. By $\eta, \epsilon \in (0,1)$, the right-hand side is negative. By Assumption 2 and $\theta \in (0,1)$, the left-hand side is positive. The proof follows. ■

It is also possible to prove that κ_λ^r is monotonically decreasing in u for a feasible set of parameters.[4] Therefore, the point where κ_λ^r and κ_λ^ℓ cross over gives us $\tilde{u}$. This implies that if such a solution exists, it is unique.

3 Empirical and Numerical Results

In this section, we present our numerical results and provide empirical support for the predictions of our model. Our model does not have an analytical solution. We run a fixed point algorithm, as

[4] The proof is available upon request. Section 3 gives one set of parameters which yield a unique solution to the planning problem.

described in Section 2.5, to compute the balanced growth path. First, in the following subsection, we discuss our theoretical findings in light of a simple simulation exercise based on a standard parameterization. Then, we describe the data we use in our empirical analysis and we report our results along with the estimation procedure. We conclude that the numerical results and the estimates are consistent.

3.1 Numerical Results

The main hypotheses of this paper are that (1) the degree of increasing returns to human capital in traditional technologies has been falling over time and (2) population growth rates and the degree of increasing returns are in positive correlation. Given these hypotheses, we ask whether sustained economic growth and declining population can coexist. In this subsection, we verify that the second hypothesis holds given the first one. Moreover, our model predicts that sustained economic growth and declining population can coexist. The experiment we perform here is to evaluate what would happen to the population growth rates and the growth rates of per capita consumption when we exogenously feed the model with a declining sequence for the parameter η which represents the increasing returns to human capital.

Our parameterization is in line with the literature. For σ, we follow the Barro-Becker tradition and set it to be less than one ($\sigma = 0.85$). That is, we stick with the complementarity assumption. Remember that the Assumption 1 imposes the restriction $0 < \epsilon < \sigma$. In line with this restriction and following Becker, Murphy, and Tamura (1990), we choose $\epsilon = 0.25$. We set $\beta = 1$, but this really does not affect the steady state solution in our model. For pure altruism, we choose $\alpha = 0.09$. Since we solve our model in a continuous-time setting, the pure altruism parameter in Becker, Murphy, and Tamura (1990) is equal to $1/(1 + \alpha)$. Note that $\alpha = 0.09$ corresponds to a slightly higher degree of pure altruism than Becker, Murphy, and Tamura (1990) assumes in their model. We pick this higher value to make sure that the transversality conditions (2.9) and (2.10) are satisfied. Our choice of $\gamma = 0.07$ is consistent with the calculations reported in Lucas (1988). We pick $\theta = 0.55$ to reflect the notion that traditional production technologies in the modern world are human capital intensive.

Figure (3.1) reports the results of the experiment. It shows that, as the degree of increasing returns to human capital in traditional production technologies decline, population growth rates fall and growth rates of per capita consumption follows a U-shaped pattern. When η falls, the economy switches from traditional technologies that need young workers to other human capital oriented production processes supporting an older population.

This shift produces two relationship patterns between population growth rates and the growth rates of per capita consumption. Initially, they move down together at relatively higher levels of η. Then, as η falls further, they start moving in opposite directions. These two distinct patterns

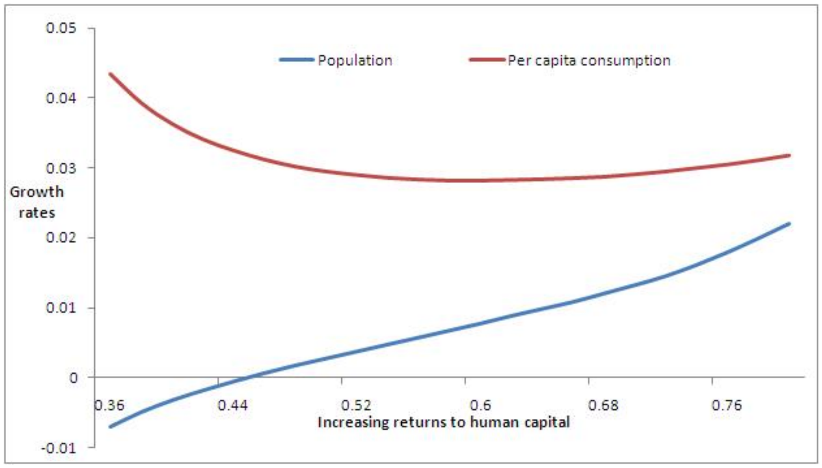

Figure 3.1: The effect of a decline in the degree of increasing returns to human capital on the growth rates of population and per capita consumption. We set $\sigma = 0.85$, $\gamma = 0.07$, $\epsilon = 0.25$, $\beta = 1$, $\alpha = 0.09$, and $\theta = 0.55$.

reconciliates two different views in the literature. While the traditional growth literature argues that the latter part must hold in the developed world, the literature linking economic growth to formation and population trends in urban areas argues that the former part must be true. Our model accomodates these two views. We argue that when increasing returns to human capital are high, growth rates of population and per capita consumption move in the same direction. The degree of increasing returns starts falling for some reason—maybe due to congestion or diffusion of knowledge to less populated areas. After a certain threshold, growth rates of per capita consumption pick up, but the population growth rates keep falling and turn negative eventually. Such a view is consistent with a strand in the urban economics literature, which mainly argues that city formation patterns follow an inverse U-shaped evolution.

These results supports the coexistence of declining population and sustained economic growth. The main idea is the substantial shift from labor-oriented production patterns to less labor-oriented production patterns. This shift is endogenous and emerges as a reaction to the decline in the degree of increasing returns to human capital in traditional production. Next we provide empirical support for our claim.

3.2 Data and Estimates

In this subsection, our ultimate purpose is to perform an estimation based on the functional form of the production function we propose in Section 2.2. To perform the empirical analysis, we need time series data on GDP, population, aggregate level of human capital, and aggregate hours of work. We obtain population, aggregate hours of work, and GDP data from the Total Economy Database of the Groningen Growth and Development Centre. We construct the aggregate human capital series based on the educational attainment data from Barro and Lee (2010).

Our focus is to provide evidence in support of the results underlined in Proposition 1. Specifically,

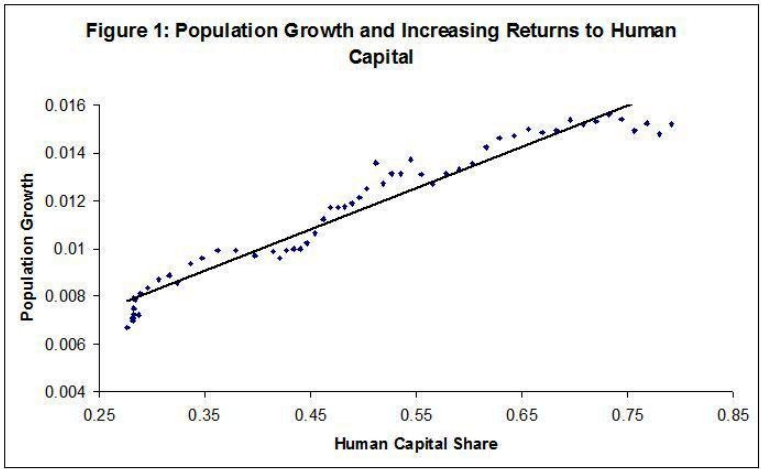

Figure 3.2: The regression results for the relationship between population growth and the increasing returns to human capital.

we want to estimate $\theta + \eta$ for a large number of countries and, then, examine the correlation between these estimates and the growth rates of population from a cross-section, time-series and panel perspectives. Provided that Proposition 1 is supported by the data, our expectation is that estimates of $\theta + \eta$ are negatively correlated with population growth rates. We perform two different exercises for the examination of this hypothesis.

3.2.1 First Pass

In our first empirical exercise, we estimate the following equation[5] within a panel-data setting[6]

$$\ln(Y_{i,t}) = \beta_{1_t} + \beta_{2_t} \ln(N_{i,t}) + \beta_{3_t} \ln(h_{i,t}) + \beta_{4_t} \ln(K_{i,t}) + \epsilon_{i,t} \tag{3.1}$$

where $Y_{i,t}$ stands for the real GDP for country i, in year t, $K_{i,t}$ is the aggregate level of physical capital[7], $N_{i,t}$ is the aggregate hours of work, and $h_{i,t}$ is the aggregate level of human capital. As one can notice from the subscripts of the coefficients, we want to allow the coefficients to vary over time. To obtain the time-variant estimates of β_{3_t}, we recursively estimate the regression equation (3.1) using panel least squares several times, each with a different time span.

In Figure (3.2), we draw the growth rate of the total population in our whole sample consisting of 50 countries against the recursive OLS estimates of β_{3_t} from 1960 to 2009. As evident from the figure, there is a strong positive correlation between estimates of β_{3_t} and the population growth rates, i.e., our first pass in the empirical analysis provides support for the hypothesis that a stronger increasing returns to human capital is associated with a higher rate of population

[5]Notice that the production function specified in the model section doesn't employ capital. In this section however, we include amount of aggregate capital among the explanatory variables when estimating the production function. We should mention that estimations and results are not prone to the inclusion or exclusion of capital. Estimation results without capital are available upon request from the authors.

[6]Our panel consists of 50 countries in total and has a time span of 60 years from 1950 to 2009.

[7]We obtain capital stock series using the well known perpetual inventory method and a standard capital accumulation equation. Data on investment is obtained from Penn World Tables and the Worldbank.

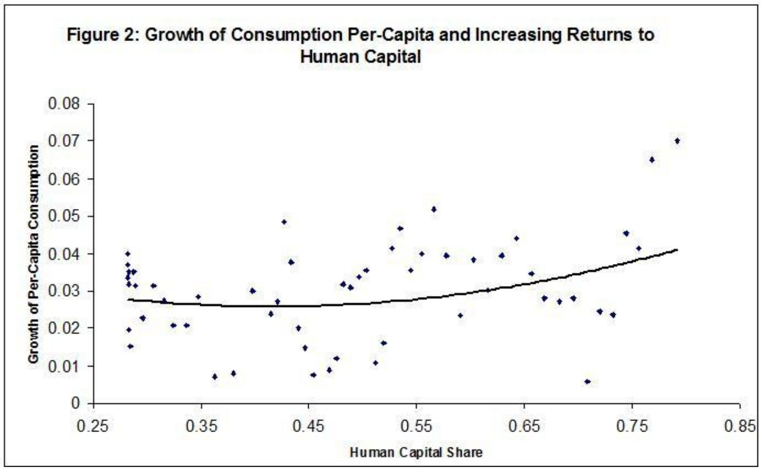

Figure 3.3: The relationship between per capita consumption growth and the increasing returns to human capital.

growth. Notice that the relationship between the growth rates of per capita consumption and the degree of increasing returns to human capital displays a U-shaped pattern as suggested by the model (see Figure (3.3)).

3.2.2 Second Pass

In addition to the simple analysis we perform above, we conduct a more detailed analysis which provides us much stronger support for our hypothesis. This analysis basically consists of two steps. In the first step, we estimate the following regression equation for each of the 50 countries in our data set over the period from 1950 to 2009:

$$\ln(Y_t) = \beta_{1,t} + \beta_{2,t}\ln(N_t) + \beta_{3,t}\ln(h_t) + \beta_{4,t}\ln(K_t) + \epsilon_t. \tag{3.2}$$

Again, Y_t stands for real GDP in year t, N_t is hours of work, K_t is the aggregate physical capital, and h_t is aggregate level of human capital of that particular country. Estimating equation (3.2) for 50 countries using recursive OLS allows us to obtain estimates of β_{3_t}, $\hat{\beta}_{3_t}^{[i]}$, for each of these countries. In the next step, we use these estimates to examine the relationship between growth rate of population and the increasing returns to human capital in a panel setting. The regression equation that we estimate in this step is the following:

$$\kappa_{i,t} = \alpha_0 + \alpha_1\hat{\beta}_{3_t}^{[i]} + \sum_{k=2}^{n}\alpha_k X_{k_{i,t}} + \theta_i + \gamma_t + \nu_{i,t} \tag{3.3}$$

where $\kappa_{i,t}$ is the growth rate of population for country i, in year t, $X_{k_{i,t}}$ are the other explanatory variables in addition to $\hat{\beta}_{3_t}^{[i]}$, and θ_i, γ_t are the country and period fixed effects, respectively. Finally, $\nu_{i,t}$ is the error term. The results of these panel regressions are reported in Table (1).

As evident from Table (1), in all the regressions we report, the estimates of the coefficient of interest (α_1) are positive and significant. In total, we run 4 panel regressions. The first three

Dependent variable: Population Growth

	Panel-OLS	Panel-OLS	Panel-OLS	GMM
Increasing Returns	0.81***	1.12***	1.69***	1.37***
	(3.99)	(2.82)	(2.57)	(3.02)
GDP		-0.24*	-0.52***	-0.31***
		(1.71)	(3.28)	(3.42)
Population Growth(-1)			-0.02	0.21*
			(-0.05)	(1.80)
GDP(-1)			-0.52***	-0.25**
			(-3.17)	-(2.05)
R-squared	0.02	0.02	0.04	0.08
Observations	2221	2221	2164	2164
F-Test		6.88	11.56	
Hansen J-Test				0.08

Table 1: Population growth and the degree of increasing returns to human capital. All panel regressions include year and country fixed effects. Robust t-statistics are reported in parentheses below coefficient estimates.

of the panel regressions report OLS estimates with different control variables in each. Finally, in the last column, we report results from a dynamic GMM estimation using the Arellano-Bond estimator to take into account the serial correlation, heteroscedasticity, and endogeneity problems that may occur in such dynamic settings.[8] The fact that the estimate of α_1 is significantly positive in all types of regressions provide strong support for Proposition 1.

4 Concluding Remarks

The demographic transition that reversed the strong population growth trend that the developed world experienced during the early stages of industrialization and brought about lower fertility rates has been the subject of intensive research in the literature. The literature has been successful in bringing alternative explanations for this transition. However, once we get to the modern advanced economy, the determinants of fertility and their interaction with macroeconomic forces are not well understood.

This paper argues that the decline in the degree of increasing returns to human capital in traditional production technologies might have led the economies to switch their productive efforts from labor-oriented technologies that require a constant creation of young workforce toward technologies that support an aging population structure. We demonstrate that, in such an environment, sustained economic growth and declining population can coexist. In support of our theoretical claim, we provide robust empirical evidence that increasing returns to human capital in traditional production technologies have been falling over time.

Our results suggest that a declining population is not a worry for modern economies, if en-

[8]Here we use aggregate level of human capital as an instrument for $\hat{\beta}^{[i]}_{3_t}$.

dogenously induced mechanisms are sufficiently effective. Such a mechanism will shift the modern economies from input-driven technologies toward gains-in-efficiency type production models. Our paper shows theoretically and empirically that the decline in the degree of increasing returns to human capital in labor-oriented production technologies may generate such an endogenous shift.

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