MIDAS Volatility Forecast Performance Under Market Stress: Evidence from Emerging and Developed Stock Markets*

C. Emre Alper[†] Salih Fendoglu[‡] Burak Saltoglu[§]

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Abstract

We explore weekly stock market volatility forecasting performance of a univariate MIDAS volatility model based on squared daily returns *vis-á-vis* the benchmark model of GARCH(1,1) for four developed and ten emerging stock markets. We compare the out-of-sample forecasting performance of the MIDAS model during the financially turbulent year 2008. We show evidence that MIDAS model produce better weekly volatility forecast than the GARCH model based on the test suggested by West (2006). MIDAS model could not generate a superior forecasting precision during more tranquil period.

Keywords. Mixed Data Sampling regression model; Conditional volatility forecasting; Emerging Markets.
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†Department of Economics and Center for Economics and Econometrics, Bogazici University, Istanbul 34342, Turkey. E-mail: alper@boun.edu.tr.

‡Department of Economics, University of Maryland, College Park, MD 20742, U.S.A. Email: fendoglu@econ.umd.edu

§Corresponding author. Department of Economics and Center for Economics and Econometrics, Bogazici University, Istanbul 34342, Turkey. E-mail: burak.saltoglu@boun.edu.tr.

1 Introduction

Following the seminal papers of Engle (1982) and Bollerslev (1986), volatility forecasting has become an extensive area of research. The different directions to which this research developed are investigated and portrayed in Granger and Poon (2003). Within the volatility modeling context, Mixed Data Sampling (MIDAS) model is introduced by Ghysels *et al.* (2004, 2005, 2006a,b), which provide ground to study parsimoniously parameterized regressions using data sampled at different frequencies. More recently, Ghysels *et al.* (2009) have studied multi-period ahead forecasts of volatility using MIDAS methodology, where they conclude that MIDAS forecasts perform well at long horizons and the model dominates all other approaches at horizons of 10-days ahead and longer.[1] Huimin *et al.* (2009) employ the MIDAS regression as well as the Heterogeneous Autoregressive (HAR) regression to predict realized range-based volatility of *S&P 500* index and compare them with the implied volatility model of VIX. They conclude that the implied volatility model (VIX) has more powerful explanatory ability than the former.

While a majority of the previous financial empirical studies utilizing the MIDAS model have used the U.S. equity return data, there seems to be very few comparative studies analyzing Emerging Market equity return data. Given the global integration of international financial markets as well as the different nature of emerging markets, the way these recent models fare in emerging countries is an interesting issue. Furthermore, even though most risk managers, options traders, portfolio managers and bank regulators frequently use long horizon measures of volatility forecasts, most papers study short term volatility forecasting such as daily volatilities. Particularly, internal risk management models and banking regulations require 10 days ahead of volatility forecasts. This paper aims to fill these gaps in the empirical volatility modeling literature. In particular, we assess the relative forecast performance of weekly MIDAS model *vis-á-vis* the benchmark GARCH(1,1) model. A systematic comparison of weekly MIDAS volatility forecasts has, to our knowledge, not been conducted before. Furthermore, a cross-country study of weekly volatilities of four developed and ten emerging stock markets has been performed.[2] In order to assess how MIDAS volatility forecasting fare under market turmoil, the sample is divided into two periods. The out-of-sample forecasting period of September 15, 2006 - August 3, 2007 is used to represent the tranquil period. For the turmoil period, we choose the forecasting period of December 15, 2007 - December 19, 2008. In order to rank weekly volatility forecasts on a continuous basis, the testing procedure proposed by West (2006) is used, minimizing the Mean Squared Prediction Error (MSPE) as the objective. Hence, the following contributions have been made in the context of volatility forecasting literature: First, under market stress, MIDAS weekly volatility model produces a much better forecasting precision than that of GARCH(1,1) model. It should also be emphasized that

[1] Even though higher frequency application of the MIDAS method received more emphasis in the literature, the MIDAS method also offers a more general analytical framework for monthly or even quarterly macroeconomic data. Ghysels *et al.* (2007) analyze the U.S. commercial real estate market within the MIDAS context. Clements and Galvao (2006) study forecasts of the U.S. output growth and inflation in the context of mixed sampling context. Hogrefe (2007) employs a study on data revisions of GDP within a mixed frequency sampling approach. Kotze (2007) uses MIDAS regressions for inflation forecasting with high frequency asset price data.

[2] We utilize daily/weekly data for two main reasons. First, intra-daily stock data for emerging markets was simply unavailable. Second, as we focus on weekly return series, we provide additional evidence on how MIDAS regression model fares under relatively less frequent samples.

MIDAS has a better precision both on developed and developing countries' volatility. This improvement is mainly driven by the use of higher frequency (daily volatility) information for forecasting weekly volatility, which is the major appealing feature of MIDAS model. For the tranquil period, we do not observe a significant improvement of MIDAS over the GARCH model.

The rest of the paper proceeds as follows. Section 2 describes the methodology. Section 3 provides the data diagnostics and the empirical results, and Section 4 concludes.

2 Methodology

The linear univariate MIDAS regression model is represented with the following econometric specification:

$$Y_t = \alpha_0 + \alpha_1 \sum_{k=0}^{k^{max}} B(k,\theta) X_{t-k/m}^{(m)} + \varepsilon_t \tag{1}$$

where Y and $X^{(m)}$ are one-dimensional processes, $B(k,\theta)$ is a polynomial weighting function depending on both the elapsed time k and the parameter vector θ, and $X_t^{(m)}$ is sampled m times more frequent than Y_t. Accordingly, the MIDAS model enables one to explore the power of $X_t^{(m)}$ in predicting Y_t where the former has a higher sampling frequency than latter. For instance, with t denoting a 5-day weekly sampling and $m = 5$, equation (1) resembles a MIDAS regression of weekly data (Y_t) on past k^{max} daily data (X_t). Thus, MIDAS regression model is able to offer a gain in efficiency by exploiting information hidden in the higher frequency data through an optimal weighting scheme.

Other appealing aspects of using the MIDAS regression model can be given as follows: (i) The polynomial $B(k,\theta)$ is parameterized in a parsimonious and flexible manner by a low-dimensional vector θ. (ii) The MIDAS model does not necessarily involve autoregressive scheme, that is, one can include any X_t that is expected to have a power to predict Y_t. Moreover, X_t can involve more than one regressor each having different sampling frequencies. (iii) The MIDAS regression model is not confined to a linear univariate framework, but can be extended to a non-linear and/or a multivariate setting (Ghysels *et al.* 2004, 2005, 2006a,b).

In this paper, we primarily focus on forecasting one-week-ahead realized volatility based on individual squared daily returns. In particular, we use the following linear univariate MIDAS specification:

$$V_{t+1,t} = \alpha_0 + \alpha_1 \sum_{k=0}^{k^{max}} B(k,\theta) \left[r_{t,t-k/m}^{(m)} \right]^2 + \varepsilon_t \tag{2}$$

where $m = 5$, $k^{max} = 50$, and t denotes weekly sampling. $r_{t,t-k/m}^{(m)}$ is the k^{th} lag of daily stock returns, where $r_{t,t-1/m}^{(m)}$ is defined as $[log(P_t^{(m)}) - log(P_{t-1/m}^{(m)})]$ with $P_t^{(m)}$ referring to daily closing value of the stock market and $P_{t-j/m}^{(m)}$ denoting the closing value of the stock market for the $m-j^{th}$ day of the week. $V_{t+1,t}$ is a measure of (future) volatility such as realized volatility where $RV_{t+1,t} = \sum_{k=1}^{5} r_{t-k}^2$. [3] Accordingly, equation (2) specifies how the previous 50 individual daily squared returns should

[3]See Ghysels *et al.* (2009) for more details.

to be weighted in predicting next week's realized volatility. Notice also that realized volatilities are based on *non-overlapping* consecutive m days which potentially avoid autocorrelation in the estimated disturbances. In order to be able to compare the MIDAS model with the benchmark GARCH model, we did not employ a non-autoregressive or a multivariate MIDAS model.

There are various alternatives for the polynomial specification, $B(.)$, among which we focus on the beta lag polynomial, following Ghysels *et al.* (2006a). In essence, beta polynomial appears to be not only parsimonious but also flexible and performing well in our in-sample forecast experiment, compared to other polynomial specifications including exponential almon lag or step functions.[4] Specifically, the beta polynomial parameterizes the weights $B(k, \theta)$ through a low-dimensional parameter vector $\theta = (\theta_0, \theta_1)$ and is specified as:

$$B(k, \theta) = \frac{f(k/k^{max}; \theta_0, \theta_1)}{\sum_{k=1}^{k^{max}} f(k/k^{max}; \theta_0, \theta_1)} \tag{3}$$

with

$$f(x, \theta_0, \theta_1) = \frac{x^{\theta_0 - 1}(1-x)^{\theta_1 - 1}\Gamma(\theta_0 + \theta_1)}{\Gamma(\theta_0)\Gamma(\theta_1)}$$

where $\Gamma(.)$ is the conventional Gamma function. The Beta polynomial, besides being parsimonious and flexible, has two other important characteristics: First, it provides non-negative weights which (almost) ensures nonnegativity of the estimated volatility. Second, the weights it offers sum up to one so that the slope parameter β_1 is identified. Given the smoothness of estimated weights, we estimate the MIDAS parameters through non-linear least squares, among other procedures.

The benchmark model based on which we assess the out-of-sample forecast performance of the above-specified MIDAS model is ARMA(1,1)-GARCH(1,1) with Gaussian disturbances.[5] In particular, we first construct friday-to-friday weekly return series (denoted by $\widetilde{r}_t$), and estimate

$$\widetilde{r}_t = b_0 + b_1 \widetilde{r}_{t-1} + b_2 \varepsilon_{t-1} + \varepsilon_t \tag{4}$$

$$E_{t-1}(\varepsilon_t^2) = h_t = a_0 + a_1 h_{t-1} + a_2 \varepsilon_{t-1}^2 \tag{5}$$

Then, denoting the predicted value for h_t with $\widehat{V}_{t+1}^G$ for all t, the forecast error for the GARCH model for the observation $t+1$ is computed as $e_{G,t+1} = RV_{t+1,t} - \widehat{V}_{t+1}^G$. Similarly, denoting the predicted value for $RV_{t+1,t}$ as $V_{t+1,t}^M$, the forecast error for the MIDAS model is $e_{M,t+1} = RV_{t+1,t} - V_{t+1,t}^M$.

It is worth noting that our specification for the two models allows us for such a comparison since, first, we exploit a linear univariate autoregressive MIDAS regression model rather than a non-linear/multivariate/non-autoregressive MIDAS. Second, as the GARCH(1,1) model can capture a large number of past shocks, we set a relatively high k^{max} in the MIDAS model, i.e. $k^{max} = 50$ by following Ghysels *et al.* (2006a).[6] Moreover, we restrict our attention to $\theta_0 = 1$ and estimate $\theta_1 > 1$ so

[4]The in-sample forecasting results under different polynomial specifications are available upon request.

[5]GARCH(1,1) specification is also used by Ghysels *et al.* (2009). In general, other GARCH specifications do not perform better than simple GARCH(1,1) model.

[6]In essence, our in-sample as well as out-of-sample forecast experiments show that lags beyond 50 does not convey significant information in predicting next period's realized volatility.

that our MIDAS model specification is not over-parameterized *vis-á-vis* the benchmark model to avoid penalizing the latter. We next present our forecast evaluation procedure in detail.

Forecast Evaluation. A common criterion to compare out-of-sample forecast accuracies of competing models is to choose the model that provides a smaller mean, mean-absolute or mean-squared prediction error, among other loss measures. Improving upon this naive criterion, Diebold and Mariano (1995), West (1996, 2006), and McCracken (2000, 2004) question the adequacy of such a procedure, and provide formal tests of equal forecast accuracy between (nested or non-nested) models for a wide variety of loss measures. In this paper, we formally compare the one-week-ahead forecast performance of the MIDAS model specified above with the benchmark ARMA(1,1)-GARCH(1,1) model, based on the Mean Squared Prediction Error (MSPE) as the loss measure. Moreover, we pursue a recursive scheme with a fixed start date, that is, we successively expand the prediction sample on a one-week-step basis. We next introduce our forecast comparison procedure in detail by first conforming our notation to West (2006).

Suppose the sample consists of $T+1$ observations and let one use the first R observations for the first roll of estimation and leave the remaining P observations for forecast evaluation such that $R+P=T+1$. Under the recursive scheme in particular, one first uses the first R observations to predict the observation $R+1$, and then uses the first $R+1$ observations to predict the observation $R+2$, and after successive rolling, one finally uses the first T observations to predict the observation $T+1$.

Let $e_{G,t}$ and $e_{M,t}$ denote the one-step-ahead population forecast errors under the GARCH and MIDAS models respectively. Also, let the difference between the squared forecast errors be $f_t \equiv e^2_{G,t} - e^2_{M,t}$. Then denoting the sample counterpart of the variables with a "$\widehat{\ }$", we have $\widehat{f}_t = \widehat{e}^2_{G,t} - \widehat{e}^2_{M,t}$. Moreover, let $\overline{f}^* \equiv \frac{1}{P}\sum_{t=R}^{T} f_{t+1}$ and $\overline{f} \equiv \frac{1}{P}\sum_{t=R}^{T} \widehat{f}_{t+1}$ denote the sample counterpart of $\overline{f}^*$.

Diebold and Mariano (1995), casting their analysis in terms of our setting, propose a simple t-test for the null hypothesis of equal population MSPEs, that is,

$$H_0 : E[f_t] \equiv E\left[e^2_{G,t}\right] - E\left[e^2_{M,t}\right] = 0$$

against the alternative that $E[f_t] > 0$. In particular, letting $\widehat{V}^* \equiv \frac{1}{P}\sum_{t=R}^{T}\left(\widehat{f}_{t+1} - \overline{f}\right)^2$ be a consistent estimate for the long-run variance of f_{t+1}, i.e. $V^* \equiv E\left[f_t - Ef_t\right]^2$, and $(e_{G,t}, e_{M,t})$ being independently and identically distributed, the t-statistic for the specified null would be

$$\frac{\overline{f}}{\left[\frac{\widehat{V}^*}{P}\right]^{\frac{1}{2}}}$$

where the inference can be done via using standard normal critical values. Nevertheless, since the prediction errors, $e^2_{G,t}$ and $e^2_{M,t}$, are contaminated by error due to parameter estimation for each model, $\widehat{V}^*$ is not necessarily a consistent estimate for V^*. Yet, since the GARCH and the MIDAS models are non-nested, one may resort to 'asymptotic irrelevance' to circumvent this problem. In particular, under 'suitable' circumstances, $\overline{f}$ has the same asymptotic distribution as of $\overline{f}^*$ implying that errors due to estimating model's parameters do not pollute the results of the aforementioned standard procedure.[7] These suitable circumstances include the following: if a relatively large sample (sufficiently large R) is used to estimate the model parameters, then the errors that pollute the standard inference will be of less importance, i.e. as $\frac{P}{R} \to 0$. West (2006) points out that a useful threshold can be $\frac{P}{R} < 0.1$, based on which we choose the forecast sample periods in our experiments. Next section presents our data set and empirical results in detail.

3 Data and Empirical Results

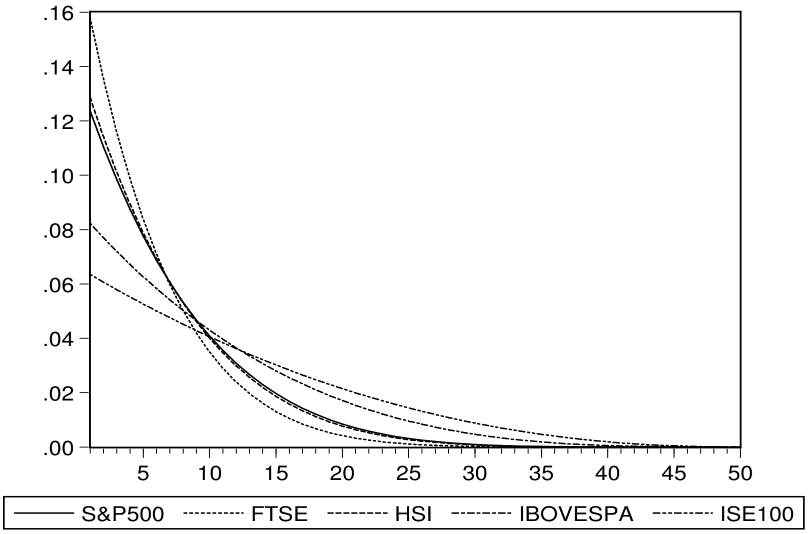
Our data set consists of daily/weekly stock returns of four developed and ten emerging market economies. In particular, we study *S&P500* (the U.S.), *FTSE* (the U.K.), *DAX* (Germany), and *NIKKEI* (Japan) among developed economies; and *BSE30* (India), *HSI* (Hong Kong), *IBOVESPA* (Brazil), *IPC* (Mexico), *ISE100* (Turkey), *JKSE* (Indonesia), *KS11* (South Korea), *MERVAL* (Argentina), *STI* (Singapore), and *TWII* (Taiwan) among emerging market economies. The stock market indices are obtained from Bloomberg. The indices for each stock market are daily closing values for the period between Jan 5, 1998 (Monday) and December 19, 2008 (Friday). The 'missing' observations due to fixed or moving holidays are replaced by the most recent available observation to achieve uninterrupted series of observations. We did not transform the data into a common currency (e.g. U.S. Dollars) to avoid potential problems based on fluctuations in cross-country exchange rates or deviations from relative purchasing power parity. For our first forecast experiment, we initially use the period Jan 5, 1998 - Dec 12, 2007, and predict the remaining weekly realized volatilities on a one-step-ahead basis. For the second experiment, we initially use the period Jan 5, 1998 - September 12, 2006, and predict the realized volatilities for a rather tranquil period of September 15, 2006 - August 3, 2007. Our aim for choosing such forecast samples is to explore whether the MIDAS model fares better in more volatile sample periods.

The diagnostics for daily return data for the whole sample are provided in Tables 1 and 2. We first present the data diagnostics for each country in Table 1, and then provide the average values of descriptive statistics for each group of countries in Table 2. The main appealing characteristic of emerging stock markets is that they provide higher returns at an expense of higher volatility, and exhibit higher fluctuations in the realized volatility. Developed markets show higher persistence in returns and realized volatility. Moreover, we observe higher within-week daily volatility fluctuations for both sets of countries for the year 2008 compared to the tranquil period of 2006-2007 (see Table 3). Hence, since the MIDAS model exploits the fluctuations in the higher frequency data optimally, one can expect the MIDAS model to outperform the benchmark in our first out-of-sample experiment where we forecast the realized volatility for 2008.

We present MIDAS regression diagnostics, specifically for our first experiment, in Table 4. The table values are the mean and the standard deviation of the corresponding regression statistics across

[7] For an extended discussion, see West (2006).

Figure 1: MIDAS Weights using the whole sample



the sample rollings (which sums to 53 out-of-sample weekly observations). The MIDAS regression coefficients $\hat{\alpha}_0$ and $\hat{\alpha}_1$ are found to be positive and significant for all countries across all sample rolls. This implies that, as expected, daily squared returns contribute positively to the following week's realized volatility. The estimated weighting schemes are presented by columns 3 to 7, and it is apparent that the weights are heavily placed on the first 15 lags, and completely die out around 35-40 lags.[8] Estimated MIDAS residuals for all countries except *HSI* and *JKSE* exhibit no serial correlation at a tenth lag order. Moreover, the squared MIDAS residuals have no serial correlation for all markets but *DAX*, *IPC*, *MERVAL*, and *TWII*. Based on these diagnostics, we conclude that our specification for the MIDAS regression model is appropriate/adequate as all parameters appear to be significant, we have no serial correlation in the first and second moments of the errors for most markets, and lags toward 50 convey no significant information.

Table 5 presents our essential empirical results: the out-of-sample forecast performance of the MIDAS regression model. The first three columns provides the diagnostics for the first forecast experiment where we initially use the sample Jan 5, 1998 - Dec 12, 2007, and forecast the Dec 15, 2007 - Dec 28, 2008. The remaining three columns present our second forecast experiment, where we first use the January 5, 1998 - September 12, 2007 sample, and forecast the period September 15, 2007 - August 3, 2008. The columns for the t-statistics are the West (2006) forecast accuracy test statistic, where a positive and large value implies that the MIDAS model forecasts statistically outperform the benchmark GARCH(1,1) model.

[8] We also present in-sample weights for *S&P500*, *FTSE*, *IBOVESPA*, ISE100, and *HSI* in Figure 1, where we use the whole sample of 1998-2008.

First observation from Table 6 is that, for the tranquil period, we can not reach a conclusion regarding the superiority of forecasting precision of the MIDAS method. As given by the sixth column, MIDAS can beat the GARCH model only for the case of *BSE*. However, we draw a completely different picture for the stressed market period. The third column presents our forecast comparison results for the global turbulent period. Apparently, seven stock markets' forecasts generated by the MIDAS model are statistically more precise than the one obtained from the GARCH(1,1) model at 90% confidence. In particular, weekly MIDAS volatility forecasts obtained for *FTSE*, *NIKKEI*, *IBOVESPA*, *IPC*, *ISE100*, *KS11*, and *MERVAL* are statistically superior to GARCH forecasts at a 90% confidence. Moreover, the result is more stark for *IPC* and *MERVAL*, where MIDAS beats the benchmark GARCH model at a 95% confidence.[9] Moreover, it is interesting to note that the forecast precision of the MIDAS model increases for almost all stock markets, both for developed and emerging stock markets.[10]

Hence, we conclude that MIDAS method shows a significant improvement over the benchmark GARCH model during more volatile periods. We attribute this improvement to the flexibility of MIDAS methodology which allows the daily data to be used in forecasting weekly volatility. Hence, this additional information in the higher frequency data during turbulent market conditions significantly improved volatility forecasts. Moreover, MIDAS model's success in weekly volatility forecasting is valid for both the developed and emerging markets data.

4 Conclusion

In this paper, weekly equity return volatility forecasts generated by MIDAS model have been compared with the benchmark of GARCH for various stock markets under the recent financial turbulence. The MIDAS model has produced a better volatility forecasting performance during the stressed market period. These results are consistent in both developed and developing countries' stock market data. We conclude that making use of additional daily return information during turbulent periods in weekly volatility forecasting provides an improvement in forecast precision. However, the same conclusion could not be drawn for the tranquil period. The use of the MIDAS model with various implied volatility models has been left for future research.

[9] To ensure that our results are not driven by our specification for the benchmark model, we compare the MIDAS model against different benchmarks, e.g. GARCH with t-distributed disturbances, and EWMA, and the results are found to be very similar.

[10] The only exceptions are *BSE30* and *DAX*.

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Table 1: Descriptive Statistics for the daily return data (Jan 5, 1998 (Monday) - December 19, 2008 (Friday))

	S&P500		FTSE100		DAX		NIKKEI225	
	daily return	daily squared return	daily return	daily squared return	daily return	daily squared return	daily return	daily squared return
Obs.	2860	2860	2860	2860	2860	2860	2860	2860
Mean	-0.003	1.718	-0.0067	1.665	0.0025	2.661	-0.020	2.434
Variance	1.718	31.552	1.665	22.930	2.661	44.475	2.434	52.698
Skewness	-0.166	11.419	-0.101	9.466	0.0031	7.447	-0.260	12.124
Kurtosis	11.671	181.661	9.268	129.559	7.277	89.111	9.881	216.628
Q(5)	36.213	1117.503	76.078	1355.920	19.620	861.583	12.389	1576.486
	BSE30		HSI		IBOVESPA		IPC	
	daily return	daily squared return	daily return	daily squared return	daily return	daily squared return	daily return	daily squared return
Obs.	2860	2860	2860	2860	2860	2860	2860	2860
Mean	0.035	2.954	0.012	3.075	0.046	5.094	0.507	2.610
Variance	2.953	51.371	3.075	95.367	5.092	422.574	2.607	47.665
Skewness	-0.364	7.803	0.149	10.435	0.611	25.838	0.195	9.110
Kurtosis	6.918	105.963	11.080	153.828	17.243	947.149	7.982	133.852
Q(5)	11.944	632.781	6.173	915.480	8.998	321.737	31.061	372.603
	ISE		JKSE		KS11		MERVAL	
	daily return	daily squared return	daily return	daily squared return	daily return	daily squared return	daily return	daily squared return
Obs.	2860	2860	2860	2860	2860	2860	2860	2860
Mean	0.071	7.958	0.042	3.087	0.040	4.285	0.016	5.080
Variance	7.953	446.850	3.085	89.376	4.284	103.961	5.080	196.395
Skewness	0.010	8.181	-0.146	9.020	-0.131	6.552	-0.138	7.488
Kurtosis	8.0607	101.287	10.400	116.305	6.672	69.129	8.612	86.035
Q(5)	13.566	504.403	57.624	288.374	7.016	356.435	16.945	444.036
	STI		TWII					
	daily return	daily squared return	daily return	daily squared return				
Obs.	2860	2860	2860	2860				
Mean	0.006	2.130	-0.019	2.575				
Variance	2.130	43.349	2.574	30.558				
Skewness	-0.063	10.852	0.099	5.634				
Kurtosis	10.549	186.320	5.603	57.941				
Q(5)	24.921	605.004	26.129	281.953				

Notes: The missing market indices (due to existence of fixed/moving holidays) are substituted by the very last available index. Q(5) denotes the corresponding Ljung-Box Q-statistic for five lags.

Table 2: Group-wise Descriptive Statistics of Stock Market -Daily Data-

	Daily Return		Daily Squared Return	
	Developed M.	Emerging M.	Developed M.	Emerging M.
Mean	-0.0068	0.0298	2.119	3.885
Variance	2.119	3.883	37.906	82.255[a]
Skewness	-1.131	0.0023	10.114	8.341[b]
Kurtosis	9.524	9.312	154.240	112.296[c]
Q(5)	36.075	20.437	1227.873	423.036[d]

Notes: Developed countries' equity markets consist of *S&P500* (the U.S.), *FTSE* (the U.K.), *DAX* (Germany), and *NIKKEI* (Japan). Emerging equity markets consist of *BSE30* (India), *HSI* (Hong Kong), *IBOVESPA* (Brazil), *IPC* (Mexico), *ISE100* (Turkey), *JKSE* (Indonesia), *KS11* (South Korea), *MERVAL* (Argentina), *STI* (Singapore), and *TWII* (Taiwan). The table values are calculated simply by averaging the corresponding statistics for each group of countries. Q(5) denotes the corresponding Ljung-Box (1979) Q-statistic for five lags.

[a] averaging all but the outliers *IBOVESPA* and *ISE100*. Including the outliers yields an average variance of 152.255.
[b] averaging all but the outlier *IBOVESPA*. Including the outlier yields an average skewness of 10.091.
[c] averaging all but the outlier *IBOVESPA*. Including the outlier yields an average kurtosis of 195.781.
[d] averaging all but the outlier *HSI*. Including the outlier yields an average Q(5) of 472.280.

Table 3: The Descriptive Statistics for Daily Squared Return Data for Two Sub-Samples

	Dec. 15, 2007 - Dec. 19, 2008		Sep. 15, 2006 - Aug. 3, 2007	
	Mean	St.Dev.	Mean	St.Dev
S&P500	6.363	15.611	0.485	1.190
FTSE	5.322	12.642	0.554	1.123
DAX	5.407	14.625	0.794	1.281
NIKKEI	7.948	19.979	0.765	1.453
BSE30	7.633	13.598	1.317	3.083
HSI	9.700	23.168	1.050	1.919
IBOVESPA	10.204	23.422	1.969	4.294
IPC	4.986	11.941	1.268	2.891
ISE100	7.124	14.841	2.388	3.964
JKSE	5.714	14.048	1.140	2.208
KS11	5.735	15.048	0.571	2.048
MERVAL	7.628	20.024	1.484	4.491
STI	4.488	10.473	1.071	2.005
TWII	4.321	7.097	0.885	2.348

Table 4: MIDAS Regression Diagnostics

	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\theta}_1$	Week 1	Week 2	Week 3	Week 4	$Q(10)$	$Q^2(10)$
S&P500	1.509	3.911	7.132	0.531	0.267	0.123	0.051	0.078	0.108
	(0.655)	(0.645)	(0.511)	(0.026)	(0.001)	(0.008)	(0.008)	(0.074)	(0.176)
FTSE	1.672	3.858	9.835	0.645	0.239	0.081	0.024	0.538	0.247
	(0.275)	(0.259)	(1.353)	(0.058)	(0.012)	(0.021)	(0.014)	(0.286)	(0.184)
DAX	2.163	4.165	7.000	0.524	0.267	0.126	0.054	0.132	0.009
	(0.178)	(0.065)	(0.771)	(0.036)	(0.005)	(0.013)	(0.010)	(0.164)	(0.066)
NIKKEI	3.714	3.189	10.573	0.526	0.221	0.127	0.069	0.203	0.143
	(0.331)	(0.263)	(11.573)	(0.213)	(0.092)	(0.061)	(0.033)	(0.192)	(0.142)
BSE30	7.541	2.214	36.441	0.966	0.031	0.002	0.0001	0.146	0.751
	(0.403)	(0.231)	(7.831)	(0.045)	(0.039)	(0.005)	(0.0005)	(0.137)	(0.308)
HSI	3.334	3.673	3.334	0.271	0.206	0.158	0.120	0.022	0.740
	(0.899)	(0.408)	(0.899)	(0.099)	(0.027)	(0.013)	(0.025)	(0.049)	(0.424)
IBOVESPA	11.406	2.552	4.230	0.362	0.252	0.167	0.105	0.919	0.998
	(0.288)	(0.169)	(0.063)	(0.004)	(0.001)	(0.001)	(0.001)	(0.052)	(0.001)
IPC	3.587	3.570	9.937	0.651	0.240	0.078	0.022	0.258	0.001
	(0.261)	(0.172)	(0.747)	(0.029)	(0.008)	(0.011)	(0.006)	(0.139)	(0.001)
ISE100	16.368	2.920	3.310	0.297	0.229	0.171	0.122	0.283	0.941
	(0.230)	(0.016)	(0.088)	(0.006)	(0.002)	(0.001)	(0.001)	(0.019)	(0.007)
JKSE	5.685	2.827	4.221	0.361	0.251	0.167	0.105	0.0002	0.163
	(0.296)	(0.153)	(0.408)	(0.027)	(0.007)	(0.004)	(0.008)	(0.0009)	(0.249)
KS11	5.286	3.641	3.053	0.204	0.122	0.105	0.099	0.120	0.652
	(1.945)	(0.419)	(5.236)	(0.228)	(0.023)	(0.032)	(0.035)	(0.054)	(0.289)
MERVAL	7.566	3.442	4.362	0.371	0.254	0.166	0.102	0.558	0.000
	(0.246)	(0.085)	(0.323)	(0.020)	(0.004)	(0.003)	(0.006)	(0.101)	(0.000)
STI	2.933	3.355	4.029	0.346	0.244	0.166	0.109	0.071	0.093
	(0.215)	(0.174)	(0.805)	(0.051)	(0.010)	(0.009)	(0.015)	(0.122)	(0.126)
TWII	3.667	3.574	3.052	0.277	0.220	0.169	0.126	0.056	0.000
	(0.073)	(0.053)	(0.092)	(0.007)	(0.003)	(0.001)	(0.001)	(0.069)	(0.000)

Notes: The table values are based on the out-of-sample forecast experiment, where we first use the sample Jan 5, 1998 - Dec 12, 2007, and predict the remaining 53 weekly realized volatilities on a one-week-ahead basis. The table values are the mean and the standard deviation of the corresponding values across rolling samples. For example, the first two entries for S&P500 show that the mean of α_0 across 53 sample rollings is 1.509, and the standard deviation of α_0 across these rollings is 0.655. All the parameters are found to be statistically significant across each sample rolling, hence we have not reported any statistic for significance. The estimated MIDAS regression is as specified in eq. (2), with beta lag polynomial under $\theta = (\theta_0, \theta_1) = (1, \theta_1)$. The weighting schemes, besides θ_1, are illustrated by the columns "Week 1" to "Week 4" subsequently, where "Week 1" represents the total weight of the most recent week (or 5 days), "Week 2" represents that of the second last week (or the days 6-10), and so forth. The columns $Q(10)$ and $Q^2(10)$ represent the p-values for the Ljung-Box Q-statistic for ten lags of the MIDAS residuals and of the MIDAS squared residuals, respectively.

Table 5: The forecasting performances of MIDAS and GARCH models: Out-of-sample Mean Squared Prediction Errors

	Forecast Sample Dec. 15, 2007 (Monday) - Dec. 19, 2008 (Friday)			Forecast Sample Sep. 15, 2006 (Monday) - Aug. 3, 2007 (Friday)		
	MIDAS	GARCH	t-stat.	MIDAS	GARCH	t-stat.
S&P500	0.089	0.109	0.977	0.0009	0.0006	-0.452
FTSE	0.093	0.114	1.494^{*}	0.0009	0.0008	-0.231
DAX	0.107	0.108	0.086	0.012	0.0014	0.455
NIKKEI	0.134	0.271	1.245^{*}	0.0015	0.0016	0.081
BSE30	0.112	0.150	1.199	0.0059	0.0098	1.362^{*}
HSI	0.528	0.687	0.966	0.0020	0.0021	0.147
IBOVESPA	0.364	0.428	1.608^{*}	0.0157	0.0140	-0.222
IPC	0.067	0.107	1.826^{**}	0.0047	0.0089	0.871
ISE100	0.145	0.174	1.555^{*}	0.0436	0.0118	-5.477
JKSE	0.174	0.185	1.001	0.0032	0.0028	-0.595
KS11	0.123	0.161	1.497^{*}	0.0027	0.0009	-3.072
MERVAL	0.253	0.541	1.829^{**}	0.0148	0.0159	0.104
STI	0.073	0.098	1.237	0.0020	0.0025	0.264
TWII	0.031	0.027	-1.274	0.0033	0.0020	-0.750

Notes: The values in the table are of order 10^{-4}. Mean squared prediction errors for each methodology are based on one-step-ahead out-of-sample forecasting. The HAC t-statistics are obtained from the regression of $\widehat{f_t} \equiv \hat{e}^2_{G,t} - \hat{e}^2_{M,t}$ on a constant. The superscripts * and ** imply that the MSPE of GARCH is higher than that of the MIDAS with a significance level of .10 or .05, respectively. The corresponding critical values are 1.282 and 1.645.